

The Oxus Auloi: Interpretation and Reconstruction

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Abstract

The aulos parts retrieved from the Oxus temple at Takht-i Sangin, Tajikistan, some of them fragmentary, are studied with respect to possible reconstructions of instruments or parts of instruments and the musical evaluation of these in the light of our knowledge of contemporary music. Even though half the material may be lost, recurring patterns indicate that some of the pipes followed a specific, obviously traditional design. Apart from these simpler instruments, others exhibit three different kinds of mechanisms for pitch adjustment, two of which have so far been undocumented. The predicted pitches are perfectly compatible with a late-Classical Greek musical system that had previously been reconstructed on the sole basis of textual evidence. The Oxus auloi thus form the first material evidence for one of the rivalling musical schools of the fourth century BCE.

Keywords

Aulos - Takht-i Sangin - Ancient Bactria - Ancient Greek music - Music history - Greek modes

1 General considerations

1.1 Complete Hellenistic doublepipes?

With no less than 43 verified bone tubes, the Oxus temple yielded one of the three most extensive finds of ancient wind instruments, topped only by the tomb of Amanishakheto in Meroë and the total related finds from Pompeii, but significantly earlier than these. Judging from diameter, bore size, and the presence of fingerholes, however, not all 43 seem to belong to the same class of instrument. Particularly, fragments 004, 011, and perhaps 027 might have formed part of bone trumpets like the *sálpinx* in the Museum of Fine Arts in Boston. At a cursory glance, all the rest looks perfectly like aulos remains from the Hellenistic period found around the Mediterranean, from Mainland Greece (Megara) to Italy (Taranto) and Spain (Teruel) in the west, Egypt (Alexandria) in

¹ Litvinsky 1999; Litvinsky 2006: 471–89; Litvinsky 2010: 435–52.



Figure 1: Bulb sections Oxus 017 and Taranto inv. 12528–7. Photographs by S. Hagel, courtesy National Museum of Antiquities of Tajikistan and National Archaeological Museum of Taranto.



Figure 2: Oxus 026 with thumb and little-finger holes displaced towards the right side. Photograph by Gunvor Lindström; design by S. Hagel.

the south, and the Asian coast (Pergamon) in the east.² The similarities include the general dimensions such as diameter, bore and fingerhole size, but more specifically the connections by spigot and socket, with the latter often reinforced by a sleeve of copper alloy, and most typically the bulbous section towards the proximal end, terminating in a cone that received the reed mouthpiece (Figure 1).³ That the instruments were not only reedpipes but indeed doublepipes like those in the Mediterranean is clear from the sections with thumbholes, which are found displaced both towards the right (012, 019, 026, 036) and the left (001, 009+043), indicating the presence of both right-hand and left-hand pipes. Like in the Mediterranean pipes, thumbholes are displaced in the direction of the respective hand,⁴ as is shown by 026, the only piece to include both a thumb and a little-finger hole. Here the latter is also displaced to the right, as is necessary on instruments that demand such large finger spans (Figure 2). Due to the importance of these sections with multiple fingerholes, including the thumbhole, as well as their uniqueness on each pipe, it will be convenient to designate potential reconstructed pipes by the numbers of these 'core' pieces.

The mismatch between the numbers of left and right core sections already indicates that what was excavated cannot possibly all come together to form a few complete instruments. Similarly, each of these six core sections would have come with a 'mouthpiece' section containing at least a reed socket but probably also a bulb, of which we however have only five (010, 014, 017, 039). Of

² Conze 1903: 7–8 with pl. 1; Jiménez Pasalodos et al. 2021; Terzēs and Hagel 2022.

³ For the development of these upper ends, cf. Wysłucha and Hagel 2023.

⁴ Cf. e.g. Psaroudakēs 2008: 202.



Figure 3: The slider mechanism on 033. Photograph by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.

these, a substantial part of 010 is missing; since this piece comes from a sifting of dumped material, it may have been broken during the original excavation. The lengths and designs of the more or less complete four bulbs are so different that they can hardly have been paired (the mouth ends of the pipes belonging to the same aulos pair are generally near-identical). We must inevitably conclude that large parts of the original instruments have been lost, probably including entire pipes. On the one hand, it is difficult to see how so much material could have been missed in such a meticulous excavation; on the other, why should the temple have stored mere scraps of instruments, and how and why would the missing parts have been discarded? The partially pristine state of the extant pieces would seem to exclude conditions so unfavourable to the survival of bone as to explain the losses either, were it not for 013, which is broken and craggy. Were there some places in the soil in which bone would have decomposed? After all, as we shall see, parts that had once been joined, were found strewn around.

1.2 Mechanisms

Most of the toneholes in the fragments are more or less circular, as is typical for fingerholes. A few are elongated and rectangular. One of these, 033, could be closed by a metal plate remotely operated by a rod (Figure 3), a mechanism well known from other finds, most notably Pergamon, Megara, Lefkada, and, from a later period and in different make, Meroë.⁵

Section 034, which belongs right below 033, also features a rectangular hole, which here appears to have been equipped with a mechanism of hitherto unknown type (Figure 4). As far as I could make out, it consists of an incomplete, open ring that could slide up and down, opening the hole or closing it to various degrees, and a complete ring of slightly tapering design that would be pushed over the incomplete one, wedging it firmly in place (Figure 5). In contrast to the sliders, this mechanism could not be operated remotely by the playing hand and would therefore serve to select the bass note in advance. By shutting the hole entirely (as well as the slider hole on 033), the lowest note would be emitted from the exit of 034 (or any additional 'bell' part, such as a horn, that might have gone into what seems to be a socket at its end). By uncovering the hole, in contrast, a

⁵ Conze 1903: 7–8; Bodley 1946; Kostoglou 1970: 331; Byrne 2002; Gänsicke and Hagel 2017; Hagel 2019; Terzēs and Hagel 2022.



Figure 4: Parts 034+033 with two different kinds of mechanism. Photograph by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.



Figure 5: The mechanism on part 034. Photograph by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.

substantially higher bass note would sound, whose precise pitch could furthermore be fine-tuned by adjusting the size of the opening.

A unique hole shape is found on 024, where the lateral sides of what elsewhere is a rectangle are curved inwards, so that the tips of two roughly triangular openings are connected by a relatively narrow channel, in an approximate hourglass shape (Figure 6). At a first glance, such a 'hide-shaped' design might be considered purely decorative; but if so, why had it not been more widely adopted? As we shall see below, the hole was situated at the underside of its pipe and therefore hardly visible during performance. Experiments with a 3Dprinted replica seem to indicate that the neck in the opening affects tone production negatively - the bass note appears somewhat feebler and is more difficult to play than one from a rectangular hole at a comparable position. Most probably, therefore, the shape was functional. Its two parts may have played two distinct notes, depending on whether the hole was fully open or half covered. When operated by a hypothetical slider, the



Figure 6: The 'hide-shaped' hole in 024. Photograph by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.







Figure 7: Elongated fingerholes in 001, 002 and 009. Photographs by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.

central neck would have increased the stability of the lower of those two notes with respect to small differences of plate position – as are hardly evitable when operating the slider during performance: thanks to the narrow diameter in this region, such longitudinal differences translate to considerably smaller differences in effective tonehole opening than they would on a rectangular hole.

The existence of a solitary slider with a long rod among the finds from Takht-i Sangin similarly encourages speculation that other rectangular holes, or perhaps even some round ones, had also been equipped with such a mechanism. After all, without it, the first holes below the fingered range would always sound and the full length of the pipe would remain unavailable. These considerations will be corroborated below, when we proceed to calculating likely pitches and intervals.

1.3 Elongated fingerholes

Apart from the (rect)angular holes outside the fingered range, three parts also exhibit distinctly elongated fingerholes. The three, however, differ in placement and therefore function (Figure 7). On 001, the hole for the left thumb extends longitudinally over about twice its lateral diameter. A single hole on 002 is enlarged in a roughly triangular shape towards the upper end of the instrument. Its placement close to the spigot at the distal end of the piece identifies it as an index-finger hole. The piece is astonishingly similar to a section from Taranto (Figure 8). Finally, 009, the core of another left-hand pipe, features a long but comparatively narrow middle-finger hole.

However different in detail, such elongated holes appear designed to facilitate half-fingering, in order to elicit at least two different notes from a single fingerhole. Their pitches would, however,



Figure 8: Item 002 (below) compared to Taranto inv. 12578–6 (above). Photographs by S. Hagel, courtesy National Museum of Antiquities of Tajikistan and National Archaeological Museum of Taranto.

have been very close together. For any plausible distance from the mouth of the player, none of the three elongated holes would comfortably have yielded even a semitone; instead, the intended steps seem to have been closer to quartertones. The same is true for the 'hide-shaped' hole on 024, discussed above: being confined to the bass region, the notes this hole emitted when either open or half-shut cannot have been further apart than about a quartertone.

Melodic quartertones are generally not very common, but are notoriously associated with the ancient Greek 'enharmonic' tetrachord divisions. Aristoxenus, discussing their origin not all too long before the Oxus instruments were made, regarded the enharmonic quartertones as a particularly Hellenic invention that set the music of Classical Greece apart from those of the neighbouring cultures:

ὕστερον δὲ τὸ ἡμιτόνιον διῃρέθη ἔν τε τοῖς Λυδίοις καὶ ἐν τοῖς Φρυγίοις. φαίνεται δ' Όλυμπος αὐξήσας μουσικὴν τῷ ἀγένητόν τι καὶ ἀγνοούμενον ὑπὸ τῶν ἔμπροσθεν εἰσαγαγεῖν καὶ ἀρχηγὸς γενέσθαι τῆς Ἑλληνικῆς καὶ καλῆς μουσικῆς.

(Aristox. ap. [Plut.], On Music 1135c)

Later the semitone was divided in the Lydian and Phrygian [modes]. It seems that Olympus furthered music by introducing something entirely new and previously unknown, thus becoming the founder of the good, Hellenic music.

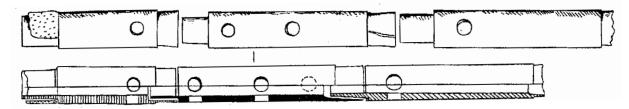


Figure 9: Sections 035+036+037 (left to right) in Litvinsky 1999: 521; Litvinsky 2006: 463 Puc. 10. Litvinsky 2010 Puc. 93.



Figure 10: Finger positions on items 035+036+037 (right to left). Photograph by G. Lindström and S. Hagel.

The characteristics of these special fingerholes therefore appear to substantiate the idea that the instruments from Takht-i Sangin establish a particularly Hellenistic musical context.

2 A well-preserved pipe (036)

2.1 Reconstruction

A few original joints have been preserved in or restored for the first publications. Most importantly this includes three sections with fingerholes, 035+036+037 (Figure 9). While 036 here forms the core with holes for the thumb, the middle and the ring finger, 035 supplies the little-finger hole, and 037 that for the index finger. In this way, we have the sizes and relative positions of a full playing hand preserved (Figure 10). Unfortunately, the upper end of 037 is broken in a way that makes it impossible to assess the depth of its socket in order to match it with some extant spigot – either of a section without fingerholes, or, more probably, of a mouthpiece section. Of the surviving parts, the bulb section 014 is the most likely candidate, judging from the socket and spigot diameters as well as the bore diameters.

At the lower end of the assemblage, a substantial part of the spigot of 035 was covered in remnants of a socket. Since it seemed that none of the other pieces lacked that much of its socket, it appeared throughout our preliminary evaluation, before the receding pandemic allowed us to examine the pieces in detail, that none of the existing parts might possibly continue the instrument downwards, destroying all hopes to establish the complete tonality of the pipe. However, a close



Figure 11: Reassigning a fragment to 008, which had been wrongly attached to 035. Photograph by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.



Figure 12: The joint between 024 and 035. Photograph by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.

inspection revealed that part of the socket fragments in fact did not belong there, but must have been glued on the spigot in a failed restoration attempt. When I was able to show that the piece in question would actually fit on 008, conservator Rustam Burchanov kindly agreed to remove it from the socket of 035 and attach it to its original place (Figure 11).

Liberated from conservatorial noise, the socket remnants on the spigot of 035 then connected perfectly to the broken socket of 024 (Figure 12), the section with the single 'hide-shaped' tonehole and therefore a natural candidate for the 'bass region' below the five fingerholes. Its hole thus comes to lie at the opposite side of the fingerhole on 035.

Part 024, in turn, fits seamlessly with 023, together with which it had been excavated. The distal end of the latter widens slightly, clearly marking the lower end of the pipe. This end was once covered in a ring of copper alloy, whose grip was ensured by a number of incised lines. While this ring is now completely gone, leaving behind only the tell-tale green discolouration on the bone, parts of it are still shown in Litvinsky's drawings (Figure 13).

⁶ They share a single excavation number, 4325, and the excavator remarks on their exemplary joining: Стык очень плотный, щель минимальная. Смыкание было сделано столь тщательно, что место стыка вообще малозаметно "The joint is very tight, the gap is minimal. The joining was done so carefully that the joint is hardly noticeable" (Litvinsky 2006: 457).

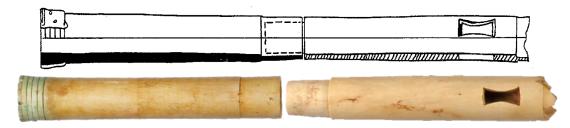


Figure 13: Sections 023+024. Above: Litvinsky 1999: 521; Litvinsky 2006: 463 Рис. 10. Litvinsky 2010 Рис. 87. Below: photographs by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.

The two 'bass sections' have been retrieved from quadrant 12, where 014, the bulb section possibly belonging to the same pipe, was also found. The three fingerhole sections in between, however, come from quadrant 7, about 5 m away. What do we learn from such a distribution? First of all, if at least one complete pipe can thus be reconstructed, it becomes more plausible that Corridor 9 once contained complete instruments, not a random selection of different sections from various instruments. Secondly, the instruments, whose sections must originally have been glued together, had already largely come apart before the event that eventually led to their burial, so that their components, only a few of which were still connected, became scattered all around. In some cases, at least, the disintegration was not merely a matter of decaying glue. As we have observed, in certain cases a socket that was still firmly glued over the spigot of the adjacent part was broken instead of coming loose. Apart from the connection between 024 and 035, this can also be observed for 018 and 025 – which were however found next to each other. We must therefore presuppose a comparatively violent event, capable of shattering thin socket walls, such as a fall from some height, followed by a period during which individual pieces were moved over several meters.

2.2 Interpretation

While the upper end of the pipe can be supplemented only speculatively, we do know the entire row of toneholes as well as their distance from the exit. Assuming that the design of the instrument incorporated a certain number of consonances, we would expect to be able to derive a plausible total effective length, including both the missing bulb section and the double-reed mouthpiece that would be inserted there. This can be done by a survey of 'harmonicity' as a function of total effective length of the pipe: if it reveals an unequivocal maximum, it is normally safe to assume that the respective configuration reflects the original musical design.⁸

The setup of the evaluation requires some fine-tuning. Following the above deliberations concerning the double function of the 'hide-shaped' hole, we must give harmonic priority to the lower of its two presumable pitches, which sounded when the upper half was closed. In ancient Greek harmonics, in any given sequence of two quartertones, only the lower (if any) could partake in the skeleton of fourths, fifths and octaves known as the 'fixed notes', which established the harmonic backbone of any melody, being related to the harmonically central $m\acute{e}s\bar{e}$ by simple consonant rela-

⁷ For an assessment of find spots, cf. Lindström in this volume.

⁸ For the method, cf. Hagel 2021.

tions. The upper of two quartertones, in contrast, would always belong to the 'moving' notes, whose precise tuning was not prescribed by Aristoxenian theory. As a consequence, we expect that the most meaningful calculations must be based on a configuration with a closed upper half of the hide-shaped hole.

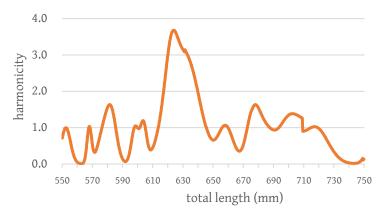


Figure 14: Harmonicity of Pipe 036 as a function of effective length in mm (Gaussian, σ =10 cents). Image by S. Hagel.

In this way, a completely automated optimisation indeed obtains

a single clear maximum, at about 624 mm total effective length (Figure 14), with five consonances

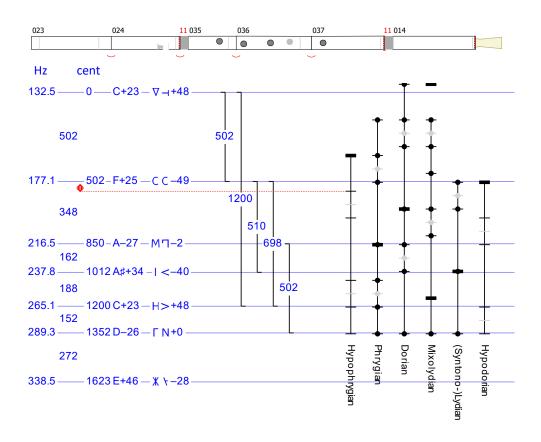


Figure 15: Plausible pitches and intervals of Pipe 036. Reed extension 34.7 mm; holes below: open except thumbhole; 'hideshaped' hole: half closed; intervals ±20 cents; reference pitch: 486 Hz; Auletic system (constant Phrygian).

From left to right: Pitch (Hz); intervals (cents); distance from bass note (cents); modern note equivalent; approximate ancient note.

'Auletic' trópoi diagram (right): circles: notes in Aristides Quintilianus' 'ancient harmoníai'; lines: additional and hypothetical notes; grey: the notes that bisect a larger interval in 'quartertones' (mesópyknoi), presumably played by half-stopping; thick lines: reference notes (mésai) of each scale.

Red line: 'hide-shaped' hole open (181.3 Hz; 545 cents above bass note: F#-35, CC-12).

within a threshold of 20 cents. The resulting pitches and intervals are detailed in Figure 15. They include a perfect octave between the exit of the pipe and the middle-finger hole, structured into a fourth and a fifth, another fourth on top of the first, and a fourth between little finger and thumb. The last is not connected to the rest of the notes by consonances, as is typical for early auloi with wide finger spans and consequently more equally spaced fingerholes that cannot include the semitones of a 'pantonic' tuning in fifths and fourths throughout. The hole that divides the octave so nicely is no other than the lower part of the 'hide-shaped' hole, whose dual function thus appears confirmed.

Based on the established plausible total length, we can now also quantify the effect of a slider over the 'hide-shaped' hole. Fully opening it would raise its pitch by about 43 cents, almost exactly a quartertone. 10

The higher of the two fourths is internally divided into about a tone in the centre with two three-quartertone intervals at each side. This reflects the typical way in which early auloi with near-equally spaced fingerholes divide the fourth. However, this fourth would not correspond to a structural tetrachord between fixed notes, which would have the tone at the top. Naturally, without the complementary left pipe, we cannot expect to read the instrument's tonality from the given pitches completely. Nonetheless, there is a way forward to an interpretation, since we can compare the pitches to those of known contemporary scales – or scale complexes, since the likely presence of a slider mechanism may imply switching between different 'modes'. Aristoxenus alludes to two existing modulating systems, one of which is more strictly based on the circle of fifths, while the other involves three-quartertone intervals between certain scales. 11 Both agree on aligning the various scales at their upper ends, as is useful for aulos players. One achieves this perfectly, while the other includes merely a single scale that starts a semitone higher than the others. The otherwise common upper end lies a fourth above the Phrygian and a fifth above the Dorian central notes (*mésai*), as well as about a minor third above the Lydian *mésē*. The former system already puts whole-tone intervals between Dorian, Phrygian and Lydian, as will later become canonical in all ancient harmonic handbooks. This aspect of continuity allows for a straightforward comparison of pitch standards: a well-established reference point, such as the pitch of Lydian *mésē* can be directly compared to the pitches of an excavated instrument. In this way it emerged, for instance, that the auloi found in Megara appear to implement precisely that system, or at least part of it. 12 The other

⁹ The respective maximum for the pipe with fully open 'hide-shaped' hole is found at 678.3 mm effective length, yielding only four consonances within the same threshold.

An equally tempered quartertone by definition measures 50 cents. When tuning the underlying semitone by alternating fifths and fourths, however, the resulting 'leîmma' of 90 cents is a bit smaller, so that bisecting it would lead to 'quartertones' of about 45 cents. The differences are hardly perceptible.

For the (partial) reconstruction of these systems, see Hagel 2000: 165–82; Hagel 2009: 378–93. It is based on the scales ('harmoníai') detailed by the Roman-Imperial writer Aristides Quintilianus (1.9, p.18–20 Winnington-Ingram), which are generally believed to reflect genuine musical structures from the Classical Period. Such systems integrate an older, presumably 'modal' concept ('harmoníai'), with the pitch differences later typical of the 'keys' (tónoi), mediated by the concept of 'styles' (trópoi). The octave species of the same names are generally considered as largely theoretical abstractions of the harmoníai.

¹² Terzēs and Hagel 2022: 40-61.

system demands more caution, since, relatively to Dorian and Phrygian, whose $m\acute{e}sai$ still maintain their canonical distance of a whole tone, its Lydian $m\acute{e}s\bar{e}$ is shifted downwards by a quartertone, sacrificing modulatory compatibility for precise alignment of the upper ends of all involved scales. This makes a comparison with later pitch standards ambiguous. One might align either the Phrygian and Dorian scales with the later system, or the Lydian: the results of the two options differ by a quartertone. In spite of its general practicality as the core note of ancient notation, the Lydian $m\acute{e}s\bar{e}$ may thus not form a suitable anchor point in a system where it is precisely the Lydian scale that stands out as the odd one. If the pitch standards followed by instrument makers remained roughly constant over some centuries at all – so far there is no evidence to the contrary – we should expect this continuity to be reflected rather in the Phrygian and Dorian, as well as, not least, in the common highest note, which is unambiguously defined between all 'modes'.

Being played by the right hand, our pipe is expected to form the higher component of the instrument. Judging from the Megara auloi, its highest fingerhole would thus have provided one or more pitches outside the underlying system, obviously conceived as belonging exclusively to the accompaniment (*kroûsis*). Only the second highest hole, the thumbhole, would then form the highest melodic pitch and thus the upper boundary of the diagram of scales.

Starting from a traditional pitch of the later Lydian $m\acute{e}s\bar{e}$ of about 245 Hz,¹³ this upper boundary would be expected to be a minor third higher, as we have said: a fourth above the Phrygian $m\acute{e}s\bar{e}$, which in turn stands a whole tone below that later Lydian $m\acute{e}s\bar{e}$, hence 245 Hz × 8/9 × 4/3 = 290.4 Hz. The corresponding actual pitch on Megara inv. 1964 was reconstructed as 294.0 Hz, and as 295.0 on Megara inv. 1965, just a tenth and a seventh of a tone respectively above our reference pitch. For the Oxus pipe with optimised harmonicity, we obtain 289.3 Hz, which is even closer. So far, our pipe behaves precisely as would be expected from a professional modulating instrument of its time.

From this potential top note, we can try to map the reconstructed early systems downwards onto the pitches of the pipe. It turns out that, among the systems reported by Aristoxenus, the more regular one, which had described the Megara instruments so well, has no explanatory value for the present pipe. Instead, the stranger system with three-quartertone distances between some of its component *harmoníai* produces excellent matches with practically all predicted pitches (Figure 15, right). Four out of five Hypodorian notes are covered (not counting those enharmonic notes that were apparently executed by half-covering fingerholes); three out of four in ('Syntono'-)Lydian; the three highest of Dorian; and four out of five in the upper range of Phrygian. Opening the presumed slider, finally, may have switched the instrument towards Hypophrygian (red line in Figure 15). Overall, this is an excellent match, considering that some pitches in the entire system are mutually exclusive wherever two neighbours are only separated by a quartertone and cannot therefore have been executed on neighbouring fingerholes. Missing pitches might have of course been supplied by the second pipe in a pair, as was here almost certainly the case with Dorian *mésē*,

¹³ For determining ancient pitch standards, see e.g. West 1992b: 273–76; Hagel 2009: 68–95.

whose pitch was also required for Lydian (and, perhaps, Mixolydian). In fact, the pitch of the ringfinger hole appears to favour Dorian above Phrygian.

All these excellent matches between predicted instrumental pitches and reconstructed scalar degrees are hardly coincidental. It appears, therefore, that this pipe from the Oxus find may represent the first archaeological corroboration of a scalar system (albeit realising only a subset of it) that had hitherto only been deduced from textual sources – just as the reconstruction of its rival system had been confirmed by an analysis of the Megara instruments. In this way, the Greek-looking pipe turns out, in all likelihood, also to have sounded Greek.

3 A modulating pipe (009)

3.1 Mechanism

The part 009, with 043 inserted, cannot be connected to any other piece with certainty; it was also found in comparative isolation, being the only piece retrieved from quadrant 10. Even so, its unique design will allow us to draw valuable conclusions. It once formed the core section of its pipe, bearing a circular thumb hole, an elongated but narrower middle-finger hole (see Figure 7) and again a circular ring-finger hole. It stands out among all aulos finds for consisting of two bone cylinders, one within the other, with matching holes. By either rotating or longitudinally moving the tubes relatively to each other those holes could be partially or totally shut. Movable despite an air-tight fit, the two components form an astonishing feat of lathe work, especially considering the delicacy of the walls that needed to be drilled and turned from such a comparatively brittle material as bone is.

While the two pieces are similar in length, the hole positions are offset, so that, when the holes are brought in alignment, the internal tube extends beyond the external at the distal end. Its lower end thus forms the spigot that connected the section to the bass part of the pipe (see Figure 17 below). On the upper end, in contrast, by falling short of the entire length of the external tube, the internal tube leaves a socket into which the spigot of the next higher section went. In this way, when the instrument was in a playable state, the inner tube was fixed (though not necessarily glued) to the lower, the outer tube to the higher end. Both parts could be separated, but without doubt the friction between the two tubes sufficed to keep them firmly together during performance. Any adjustment of the internal tube was necessarily achieved by grasping and adjusting the lower part of the pipe – and therefore possible only during a break in the performance.

On later instruments, we find instances of sections with three holes that could be closed all together by a single external sleeve of metal, in order to shift the playing position of the hand to a lower region (Figure 16). However, it is of the highest importance to recognise that this cannot have been the purpose of the Oxus piece. Closing merely the holes for the thumb, the middle and the ring fingers would be useless as long as the index-finger hole is left open to sound – but the latter is conspicuously excluded from the adjustable section. A separate (lost) mechanism to close



Figure 16: An example of a single sleeve operating three holes from the Meroë find (part 048 = Bodley 1946 pl. 3.4). Photographs by S. Gänsicke, courtesy of the Museum of Fine Arts, Boston.

it would also make no sense, precisely because there is no advantage in opening and closing index and thumb holes independently. The conclusion is inevitable that the rotatable tubes of section 009+043 were not meant for closing the three holes, but for changing their sizes and consequently their pitches: decreasing the area of a fingerhole will lower its pitch, albeit by a comparatively small amount.

By which of the two possible movements was this diminution of hole size effected, by pulling the internal tube downwards or by rotating it? The former appears a priori less likely because it would open up a gap in the smooth external surface of the instrument, exposing the

smaller diameter of the inner cylinder. It also cannot have made musical sense. When the internal tubing is pulled downwards by a few millimetres, all further toneholes below, including the exit of the pipe, are repositioned by the same amount, while maintaining their sizes. This also lowers their pitch, but by a significantly smaller amount, the less the further down the instrument they are. For instance, with a displacement of 4 mm, the circular holes of 009 are about halved. With a typical instrument length above the thumb hole, this leads to a pitch drop of about 30–40 cents for these three holes, while the little-finger hole below would be lowered by only about 15 cents, and any bass notes below, somewhat less. I cannot conceive of any musical structure that might benefit from such oddly-sized adjustments, which would inevitably destroy any harmonic relation between index, little-finger, and bass holes there might be. Almost certainly, the mechanism of 009+043 worked by rotation, just as it did in all the later metal-clad auloi.

With a rotational mechanism, we cannot a priori assume that the relative azimuthal position of the holes in both tubes matched perfectly, so that all can be fully open at the same time. One may perfectly imagine an arrangement with alternately half-closed holes and therefore varying interval sizes in between. Unfortunately, when measuring aulos fragments, it is almost impossible to determine the relative hole azimuths with precision. Normally, I place the pieces upright on a stake which penetrates the sheet of paper on which I mark the orientation of the holes; but since these are increasingly removed from the end that rests on the surface, errors of a few degrees are hard to avoid. In the present case, physically reinserting 043 into 009 might have been the simplest way, provided that this is not prevented by protective surface treatment. Being distributed between two different museums, however, the items did not even lend themselves to a direct visual comparison. Nor did we have radiographic images, which would help settle the matter, provided both are taken with identical orientation of a reference hole. Instead, I managed to prepare sufficient photographic documentation of all pieces to compose photogrammetric models with the expert help of Moisés Hernandez Cordero from the Austrian Archaeological Institute, which it was then possible to assemble either virtually or physically in the shape of 3D prints. It emerged that



Figure 17: Comparing hole azimuths of 009 and 043 by virtually assembling photogrammetric models. Image by S. Hagel.

the hole arrangements actually matched perfectly, so that the three holes would be fully open at the same time and, on rotation, laterally close by the same amount (Figure 17). However, the central hole being elongated but narrow, the same degree of rotation would have closed off a larger fraction of its surface compared to the two circular holes.

3.2 Interpretation

The positions of the adjacent tone holes, that for the index finger above, and that for the little finger below, are limited on the one hand by the extent of the items 009 and 043 themselves, on the other by the practical constraints on human fingering spans (Figure 18). The index hole must have been removed from the upper end of 009 by at least a few millimetres. The comparatively large distance between thumbhole and upper end of 009 already brings about quite a large span between index finger and thumb, so that the hole for the former was very probably located rather close to the point where the sections met. The little-finger hole, on the other hand, was necessarily drilled beyond the end of 043, which served as a spigot and has a smooth rim.

Regarding the calculation of possible frequencies for the three holes of 009+043, these are unaffected by the (closed) index hole above, because it influences all three in the same manner. The little-finger hole, in contrast, if it is left open, would slightly raise the note of the ring-finger hole; but the effective variation for different distances is practically negligible.

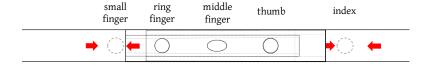


Figure 18: Possible positions for index and little-finger holes below and above 009. Image by S. Hagel.

In the following model calculations, item 022 is joined to the lower end of 009+043, which not only matches their external and internal diameters and features a hole within the desired range, but whose socket also precisely accommodates the spigot part of 043. This is no certain joint, but the only one possible within the extant cache, and a quite plausible one given the excellent physical fit. Whether more toneholes were present below plays no role for the pitches in question, because 022 itself is so long that holes beyond its lower end do not affect the notes played from the fingered holes.

Above 009+043, I have tentatively added piece 002. While this is once more the best fit available, it is apparently not perfect; unfortunately, the separation of the pieces in two museums here also prevented a direct physical assessment. At any rate, in view of the large distance between the upper rim of 009 and its thumbhole, the interval between the notes sounded by the index-finger and the thumb holes respectively must have been significantly larger than those between the holes of 009+043. If the latter were about the usual size on this kind of aulos, i.e. between roughly three quartertones and a small tone, and the index hole about the same size as the thumb and ring-finger holes, the top interval would have measured at least a minor third (assuming the closest attested distance between section rim and index hole, which measures 7.9 mm on part 040). The larger, slightly elongated hole on 002, which is further removed from the lower rim would even yield an interval of seven quartertones. However, the specific upwards enlargement of this hole (Figure 8) indicates that it was designed for half-covering and therefore played at least two distinct pitches, about a quartertone apart. As discussed above, Greek theory would always attribute harmonic primacy to the lowest pitch in a row of quartertones; this 'basic' pitch would thus reduce the interval above the thumb hole to the same minor third that a normal-sized hole close to the rim would also yield.

Can we move beyond such general considerations and specify the pitches and intervals with greater precision, establishing the likely missing upper length of pipe plus reed? For merely three adjacent fingerholes, no meaningful harmonicity maximum can be obtained. At most, one might postulate a precise fourth between thumb hole and the somewhat speculative little-finger hole on 022. This results in the pitches shown in Figure 19 in blue colour. Interestingly, but perhaps incidentally, the half-covered index hole establishes another perfect fourth, above the middle-finger hole.

To the right, in red, the frequencies are displayed that result from turning the mechanism of 009+043 by about 30°, corresponding to a pitch drop of about a quartertone. Here the upper fourth is destroyed and replaced by a perfect fifth from the top note down to the ring-finger hole.

One will observe the tiny rise in the pitch of the little-finger hole (188.9 Hz instead of 187.5 Hz), which is caused by the reduced cavity of the three holes above it. Here the calculation does not fully reflect the geometry of the instrument, because it assumes an area reduction throughout the tube wall, while in reality the internal half of the cavity, within 043, is fully present. The actual pitch difference is therefore expected to be only half that shown in the diagram, i.e. about an insignificant 7 cents instead of the calculated 14 cents.

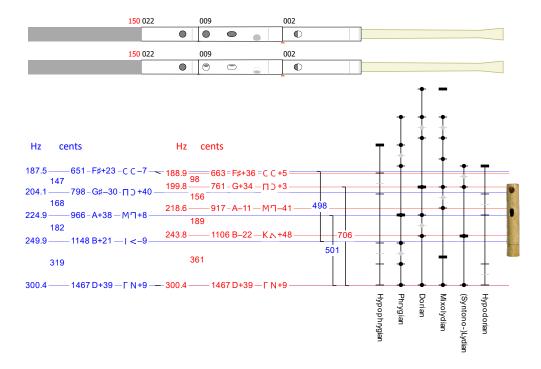


Figure 19: Pipe 009 with exempli-gratia neighbours 022 and 002, in two different states: above/blue: holes fully open; below/red: holes partially closed by rotation by 27°.

Holes below: open, thumbholes closed; reference pitch: 502Hz; Auletic system (constant Phrygian). Image by S. Hagel.

A comparison with the reconstructed pitch relations of the same system of *trópoi* as above reveals stunning coincidences if the highest notes of pipe and system are equated, as is expected for a left-handed pipe like the one under scrutiny. Both the higher and the lower variants from the three holes in 009 correspond to structurally important 'fixed' notes in the *trópoi* (*barýpykna* and *mésai*) in one scale or another. If the joint with 022 is correct, the case of the little-finger hole would however be different: while the open hole would apparently play a pivotal note in the (slightly hypothetical) Hypophrygian, any other *harmonía* would require lowering its pitch by reducing its opening with the finger. But perhaps, instead of 022, the original pipe contained a lost section with a fingerhole about 12 mm further down the pipe, which played the required pitches of the other modes and left merely the Hypophrygian note to be produced by the comparatively unstable technique of partially uncovering the ring-finger hole.

The absolute pitch of the reconstruction so far depended precisely on the hole position of 022 and the construction of a perfect fourth on its basis. The ensuing reference pitch in Figure 19 is 40 cents higher than 490 Hz, and therefore a quartertone higher than the apparent pitch standard of Pipe 036 above. However, with slightly 'less perfect' fourths, and even more so with a different little-finger section, a good coincidence with the *trópoi* system can be maintained within a certain pitch range. Pitches and intervals for an optimal fit to the 490 Hz standard are given in Figure 20.

Once more, we can only surmise what notes the higher, right-hand pipe would have supplemented. For instance, we might imagine that the 'quartertone' couples ($pykn\acute{a}$) above the thumbhole note were more easily produced using the right thumb on a hole that was drilled only

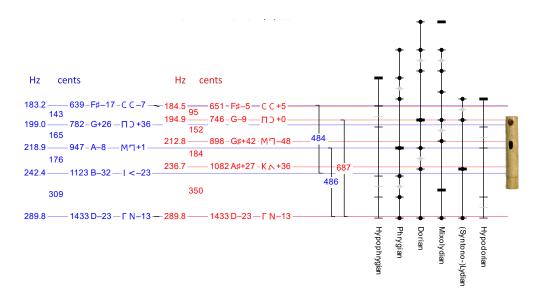


Figure 20: As Figure 19, but conforming to a reference pitch of 490 Hz. Image by S. Hagel.

about three quartertones higher, instead of the minor third we have deduced for the left-hand pipe. More probably, though, the right thumbhole may have reduplicated the highest pitch of the left pipe, while it was the middle finger that realised the *pyknón*. ¹⁵

Such a design would also accord better with that of 009+043, whose distinctly elongated middle-finger hole appears to have been produced specifically to facilitate the production of the Mixolydian and Hypodorian $pykn\acute{a}$ above the ring-finger note. ¹⁶

On the other hand, it seems we are in a position to determine the use of the rotating mechanism for accessing the various harmoniai or trópoi. With fully open holes, the aulos was set up for Hypophrygian, Phrygian, and Hypodorian. Narrowing down the openings by a quartertone would instead have given Mixolydian and Lydian (is the shared 'Lydian' element in their names a coincidence?). Dorian, finally, would apparently have availed itself of the same half-closed state, but one of its 'fixed' notes, $paramés\bar{e}$ M \square , would have required partially opening the thumb hole, at least if it was to be played at its theoretical 'fixed' pitch a fourth below the top note, $n\bar{e}t\bar{e}$ (it goes without saying that the quartertone above it is accessed by further uncovering the same hole).

The last caveat is necessary because one might be enticed to relate the present aulos design to a well-known passage, cited from Aristoxenus by pseudo-Plutarch (*Mus.* 1145d), where the author, in the course of an argument for the ubiquity of intervals that do not conform to a semitone grid, refers to a widespread habit of down-tuning certain notes by "irrational" intervals, ¹⁷ even

¹⁵ I am grateful to Chrēstos Terzēs for this brilliant suggestion.

Middle-finger holes specifically designed for half-holing are now documented on later instruments as well, both from Pompeii (Wysłucha and Hagel 2025) and from Meroë (Hagel 2019: 185).

¹⁷ In Aristoxenian context, the term 'irrationality' does not invoke the mathematical concept of not being expressible as the correlation of integers, but refers to any size that human perception cannot readily relate to established entities such as tones and their simpler fractions in harmonics, or long and short times in rhythmics.

including certain 'fixed' notes, contrary to their status in harmonic theory. 18 Especially the practice of "lowering the tritai and the paranêtai in addition to/towards" the lowered fixed notes ($\pi \rho \sigma \sigma$ ανιέντες αὐτοῖς τάς τε τρίτας καὶ τὰς παρανήτας) might, in enharmonic auletic context, be understood as an automatic, physical effect: when, as is arguably the case on our pipe, the 'fixed' note of Dorian paramésē is too low, it follows that the two next higher notes, which are produced by gradually uncovering the thumbhole, will be low as well. However, such an interpretation would meet fundamental and apparently insurmountable difficulties. Firstly, the interval in question would in theory amount to a quartertone and therefore would not fall within Aristoxenus' category of 'irrational' intervals (áloga) but merely create one of those that comprise an odd number of quartertones (perissá). Secondly, the start of the passage with its focus on down-tuning likhanoí and paranêtai only appears comprehensible in the context of diatonic music, because it would make no sense at all within Aristoxenus' conception of the enharmonic, whose standard shape already places these notes at the lowest possible pitches. On balance, Aristoxenus cannot have been thinking of instruments like ours, but was more probably referring to cithara music.¹⁹ Nonetheless the apparent design flaw of a 'lowered Dorian paramésē' may have been significant, as we will learn below.

3.3 Conclusion

Although merely a single section from one pipe, with no more than three fingerholes, may have survived, the uniqueness and importance of this find can hardly be overstated. On the one hand, the technical features, both the rotational mechanism and the elongated fingerhole, can be made functional within the same Greek harmonic system that pipe 036 also appears to reflect. This regards not only the intervallic structures, including the quartertone shifts that are so typical for this specific pre-Aristoxenian solution to combining various modal scales on a single instrument, but also the particular pitch at which they were realised. In this respect, we appear to have another 'Greek' instrument, in the sense that it shows no characteristics that cannot be explained in the context of what we know about Greek music in the Late Classical period. On the other hand, this is achieved by bone tubes with a row of fingerholes rotating within each other, a device that is so far undocumented from the Mediterranean. Was it a Bactrian invention? The possibility should not be excluded, but the scarcity of mechanical-aulos finds from the pre-Roman period should caution us against rushing to such a conclusion. After all, fragments of rotating bone mechanisms of different sorts - but arguably even greater sophistication - have been found in Megara as well as Taranto (Figure 21 and Figure 22). The general technology, therefore, was available throughout the Hellenistic world. Since it rested on two separate comparatively thin bone shells, we may also be facing a factor of archaeological bias: the typical more thick-walled aulos sections may have had much greater chances to survive unfragmented. Most importantly, we have no other find where the quar-

 $^{^{\}rm 18}~$ For a discussion of this passage, see Hagel 2009: 139–42.

¹⁹ Cf. also the usage of the term *maláttein*, 'soften', for the procedure, which at least originally refers to relaxing a string.



Figure 21: An aulos section from Taranto (inv. 12528–4) with large rectangular tonehole with rotating-tubes mechanism, operated by a metal lever inserted in the socket of the protrusion top-right. Longitudinal movement was prevented and rotary movement restricted by a pin through the external tube, running in a groove in the internal tube (right). Photographs by S. Hagel, courtesy National Archaeological Museum of Taranto.





Figure 22: An aulos section from Taranto with round tonehole with rotating-tubes mechanism, operated by a metal lever.

Longitudinal movement was prevented and axial movement restricted by a pin through the external tube running in a groove in the internal tube (right). Photographs by S. Hagel, courtesy National Archaeological Museum of Taranto

tertone shifts between different scales are reflected in a mechanism affecting the fingerholes (as opposed to the bass holes, which could be adjusted in quartertone steps on the Megara instruments).

Apart from the unique mechanism, the large interval of an undivided minor third between index finger and thumb is also noteworthy. In this respect, the pipe containing 009+043 stood also out of its colleagues in the Oxus temple. In fact, as far as we can see, the other instruments, such as that containing 026, would have distributed all five fingered toneholes within the span between index and ring finger on 009+043. In addition to its technical sophistication, our pipe thus also posed extreme demands on the player's left hand. There is little doubt this was a highly professional instrument, probably reflecting the pinnacle of the auletic art in the Late-Classical and/or Early-Hellenistic periods.

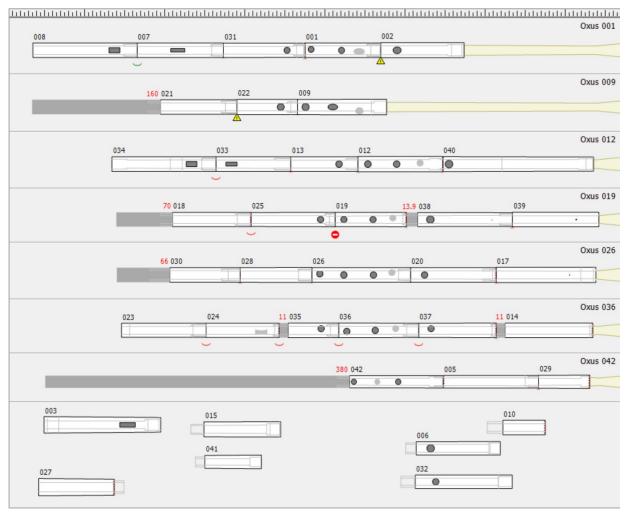


Figure 23: Partly hypothetical arrangement of apparent aulos fragments to pipe parts. Random extensions indicate lost bass sections. Screenshot from the software used for virtual experimentation. Image by S. Hagel.²⁰

4 Other instruments

4.1 Reconstruction

What can we learn from the remaining fragments? First of all, there are a few recurring patterns. Of the other right-hand sections, the three fingerholes on 019 are distributed quite similarly to those of 036, and so are the higher three of 026, which is however long enough to include the little-finger hole as well. On item 012, however, the distance between these and the thumbhole is distinctly larger. Item 042, finally, not only stands out for its slenderness, but also uniquely combines index and thumb holes on the same section, while relegating the ring-finger hole to the next lower one. Its thumb hole is drilled just opposite the others, so that the piece does not divulge its handedness.

Red brackets indicate certain joints, green, plausible ones. Yellow warning signs point to small mismatches in the measurements of sockets and spigots. The warning sign between items 019 and 025 indicates that this



Figure 24: Fingering items 026+020. Photograph by G. Lindström and S. Hagel.

Based on diameters, spigot and socket sizes, it is possible to adjoin other parts to these cores with varying degrees of certainty (Figure 23). In this way, at least the minimum number of pipes can be established that must have been involved some way or other.

All single round toneholes that are situated on long pieces close to their spigot, which invariably marks the distal end (with the possible exception of the enigmatic 027), appear to have been index holes. In addition to 037, two more of these, 020 and 038, fit respectively into the mentioned cores of 026 and 019 (cf. Figure 24),²¹ while 040, whose spigot is broken, might equally well go with 001 and 012. Another one, 002, might possibly have belonged with 009, where we have used it tentatively above. However, this is the most doubtful of the proposed joints, because the diameters of both bore and spigot of 002 appear a little too large. At any rate, even if it may not belong above 009, its fingerhole position cannot be far off, so that we may use it as a proxy for now. There remain two more presumable index-hole sections, 006 and 032, whose core pieces are obviously lost. Of course, one might try to place them in the bass region of some pipe; but on closer inspection, no likely positions can be found. In the end, it seems that, in contrast to the instruments from Megara or Pergamon, all holes below the five fingerholes were here made in rectangular shape. As a result, we seem to have the remnants of at least nine pipes.

joint is not among those physically confirmed possible during our research trip. However, this is due to the sad fact that the two pieces are currently separated between two museums. Being unable to get the entire find on the same table unfortunately prevented us from collecting important information of this kind.

²¹ One might consider 032 instead of 038; this would make no difference for the following conclusions, since the narrower hole of 032, which is situated slightly higher, plays the same pitch as the wider hole further down on 038 (the calculated difference amounts to no more than 4 cents).



Figure 25: Finger positions on items 031+001. Photograph by G. Lindström and S. Hagel.

Conversely, sections with round holes close to the socket and therefore the proximal end are candidates for little-finger holes. Five such pieces have survived, and all may belong to existing cores: in addition to 022 below 009+043, and 035 below 036, these are 013 below 012 (which were found together), 031 below 001 (Figure 25), and 025 below 019.

Item 022 can be connected to 021, next to which it was found (they have consecutive excavation numbers). Below 031, item 007 with a rectangular bass hole fits well. This, in turn, as we were able to establish, should be joined to 008 (Figure 26). Similarly, 025 is continued by 018. Item 026, which already includes a little-finger hole, may be prolonged by 028 and 030, both lacking any holes, without however reaching the end of the pipe. Finally, the spigot of 013 would match several sockets, above all that of 003, an end piece with rectangular hole. However, the resulting pipe would appear a bit short in comparison to Pipe 036 with its complete distal end, and more so to the expected lengths of the incomplete Pipes 019 and 026, and 003 is also spurious because it was allegedly found above floor 2 and might therefore belong to a different period. Instead, the compound of 034+033 terminates the tube nicely, making it just a little longer than Pipe 036.

Regarding the upper ends, item 040 is so long that, wherever it belonged, it appears to leave no space for a bulb. Presumably the reed went directly into its socket. The existence of bulbless



Figure 26: The re-established joint between 007 and 008. Photograph by S. Hagel, courtesy National Museum of Antiquities of Tajikistan.



Figure 27: Items 014 and 010, to scale. Photographs by G. Lindström, courtesy National Museum of Antiquities of Tajikistan.

auloi may contribute to explain the puzzling fact that remains of only four bulbs have been retrieved from the Oxus temple, half as many as expected, even though these feature the thickest walls (next to the thinnest). Many vase paintings from the classical period portray doublepipes just in the form of two couples of parallel lines without any curvature towards the mouthpiece. While this can be interpreted as mere negligence, it seems clear that simpler instruments made of cane could not have had bulbs. Bulbless bone pipes may therefore just continue a different aesthetic.

Of the preserved bulbs, 014, 017, and 039 are of such diverse shapes that they must have belonged to different instruments, since the bulbs of extant pairs are universally found to appear identical. Item 010, however, which preserves merely the lowest few centimetres of a bulb, may or may not have been paired with 014 (Figure 27); if not, it may be a remnant of the missing top part of the somewhat thicker Pipe 009.

Physically, 017 fits above 020, and 039 above 038 (alas, the adjacent numbers in our catalogue here do not imply proximity of find spots; we do not know from which quadrant 039 was retrieved). Pipes 019 and 026, in spite of lacking their lowest sections, thus have not only an identical structure concerning sections and fingerholes, but also identical lengths of 53.2 cm. Is this a corollary of ancient production processes or just a truly remarkable coincidence? If the former, it makes the end sections stand out, because no identical boundaries are observed anywhere else. It is however difficult to see how those bass sections would be any different. Might one entertain the possibility of exchangeable 'bells' of different lengths, functionally reflecting bass holes with pre-set mechanisms on other instruments? On the other hand, the two pipes already reach beyond all toneholes observed on the other right-hand pipes in the find. Were they instead equipped with bells of a different, more perishable, material, such as horn, which was produced in standard sizes in another workshop?²³

Judging from the various diameters, bulb 014 may have belonged into 037, but we lack crucial confirmation from the comparison of spigot and socket lengths because the socket of 037 is almost entirely broken off. We shall presently see, however, that it produces an overall upper length that is in perfect accord with the other pipes. So far, a common configuration emerges: above the item with the index fingerhole, only one more section is found, which terminates with the socket for

²² Cf. Wysłucha and Hagel 2023: 376; 381-82.

²³ For aulos bells apparently made of horn, cf. ps.-Aristot. *De audib.* 801b; 802a (ποιοῦσι τὰς φωνὰς ἀμαυράς "darken the sound"); 802b (hardened by baking; cf. 803a).

inserting the reed mouthpiece (note that item 040 actually consists of two bone sections, which were connected in the images from which we originally worked during the pandemic). But how would 042 behave in such a context? Its index finger hole being integrated in the 'core' piece, the length of tubing above it is not nearly as long as on the pieces that accommodate only an index hole. Of the extant pieces, only 005 may go above it, and though this section is as long as any, indicating that it utilises the entire usable part of the bone, it inevitably falls short of the typical distance between index finger and upper pipe end by more than 6 cm. This distance is apparently too short for an additional bulb, but too large for bridging it with an exceptionally long reed stem. Therefore, either 005 does not belong here, after all, but some shorter spacer tube below a bulb, or this pipe also had no bulb but only another, shorter cylindrical section terminating in the reed socket. Indeed we have a possible socket of such a kind: item 029 is of just the right length and appears to fit into 005, even though its bore is slightly narrower.

Bulb 010, if it was indeed a sibling of 014, would have belonged to the lost left-hand counterpart of Pipe 036, whose distinctive parts are otherwise lost. Item 003, if it belongs with the rest of the pieces, after all, might have formed its bell (within the surviving pieces, its rectangular hole is at the same time the only possible home for the long slider 051, though, with a length of 25.6 mm, the latter's plate appears still a bit too long for a hole of merely 19.4 mm). Of the remaining items, 015 is a spacer similar to 018, 028, and 030, which all range between 88.0 mm and 98.2 mm effective length; it might have formed part of the lost end of Pipe 009. The slender 041 may come from the same instrument as the index-hole section 032; on the other hand, it is almost as short as 029 and might just as well have been the uppermost section of a bulbless instrument. Item 006, with similar external diameter as 032 but apparently a somewhat wider bore might conceivably be the only remnant of its counterpart pipe.

Apart from the mysterious 027 – the apparent 'bell' with upper spigot instead of a socket, contrary to the otherwise uniform construction principle of the Oxus fragments, if it belongs to an aulos at all – we count now at least ten pipes: those around the cores 001, 009, 012, 019, 026, 036, and 042, the one terminating in 003 (and possibly starting with 010), and those with the index-hole sections 006 and 032.

4.2 Number of pipes

How many instruments were involved? The preceding survey of reconstructive options, and the often implicit refutation of vastly differing options, have proven beyond possible doubt that the instruments in question were doublepipes, with no more than five fingered holes per tube, at least sometimes augmented by angular bass holes, which are found equipped with two different types of mechanism. On the other hand, the ten or more partially reconstructed pipes do not mate easily. After all, we have only two left-hand pipes. Of these, Pipe 009 is noticeably wider than all others; since all known pairs are composed of tubes of nearly identical diameter, we cannot expect to find its counterpart among the documented fragments. Similarly, partial pipe 042 remains isolated be-

cause it is so slender. We have already stated that whatever belonged with 010 (and/or 003) may have formed the counterpart of Pipe 036. Possibly, item 032 may have been paired with Pipe 019. Of the right-hand pipes, this would leave 012 to go with Pipe 001, bringing the total of potential pairs to three. On the assumption that complete instruments had been stored in the temple, not single pipes or even fragmentary pipes, the conclusion appears inevitable that in addition to the substantial parts missing from the attested ones, the four pipes to complete the instruments containing parts 026, 009, 042, and 006 are lost (unless they contained some of the stray parts whose attribution to the attested ten is hypothetical). We will return to these questions in more detail below.

4.3 Musical evaluation

Can we elicit some musical sense from the attempted reconstructions? The most straightforward task is to compare the predicted musical characteristics of those designs which, at a first glance, resembled the unambiguously established pattern of Pipe 036 with the pitch patterns predicted for the same Pipe 036.

4.3.1 Pipe 026

As shown in Figure 28, the rows of fingerholes are musically identical on Pipes 026 and 036. For optimal congruence with the intervals of the *tonoi* diagram, the pitch level is here set a bit lower than in Figure 15 above, so that the fourth between little finger and thumb, the only interval expected to reflect one of the ancient consonances on 026, is a perfectly pure fourth (498 cents). This also creates an absolutely perfect fourth on 036, with the two ring-finger notes in between being

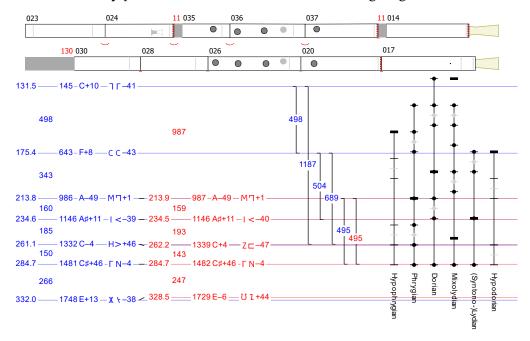


Figure 28: Pipes 036 (reed 39.3 mm) and 026 (reed 30.9 mm). Holes below: open, thumbholes closed; intervals ±20 cents; reference pitch: 479 Hz (490 Hz – 40 cents). Image by S. Hagel.

perfectly identical, the middle-finger notes differing by no more than 7 cents, and the index notes by just 12 cents. Such a stunning coincidence strongly suggests that these are examples of a fixed template. However, we are not dealing with a design template, but a genuinely musical one. While the two index-finger as well as the little-finger holes may respectively produce the same notes, they are not drilled at identical distances from their respective neighbours. The instruments seem to have been carefully tuned rather than being mechanically reproduced.

4.3.2 Pipe 019

Pipe 019, if reconstructed correctly, forms a fascinating variation of the same scheme. While the fourth between thumb and little finger is maintained, the index note is considerably higher, which is achieved not only by taking the finger stretch to an extreme, but also by enlarging the index hole relatively to the others (Figure 29). While the top note of the other pipes must thus be produced by shading the opening with the fingertip, the new treble note boosts the harmonic potential of the instrument by forming a pure fourth with the middle-finger note – and therefore probably a corresponding hole of the lost left-hand pipe, most typically the thumb hole (but see below on the possibility of a different shift between the hand positions). The higher top note also provides another opportunity: by tuning the ring-finger note only a little bit lower, it produces a perfect fifth below the treble note – which would most noticeably materialise not as subsequent pitches on one pipe, but again as a simultaneous concord between right index and, typically, left middle finger. The difference impinges on the size of the respective *pyknón* above Dorian *paramésē*, but not on the harmonic framework. As enticing as such an improvement must have been musically, the required stretch may not have been for everybody. Astoundingly, we find a very similar

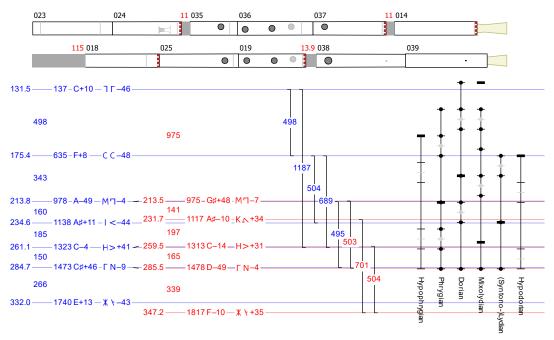


Figure 29: Pipes 036 (reed 39.3 mm) and 019 (reed 26.0 mm). Holes below: open, thumbholes closed; intervals ±20 cents; reference pitch: 480 Hz (35 cents below 490 Hz). Image by S. Hagel.

difference between the two auloi excavated in Megara, where the description of Meg2 versus Meg1 might be applied to Oxus 019 versus the two other discussed pipes one to one:

"The treble note of the whole instrument is significantly higher. Fully opened, it produces a good fourth with the thumb hole of the left-hand pipe, and a good fifth with its middle-finger hole, crucially augmenting the harmonic capabilities. The required increased finger span, however, may not have been accessible to the owner of Meg1."

(Terzēs and Hagel 2022: 60; cf. also 39).

The Megara auloi, however, do not reflect the same arrangement of *trópoi* that worked so well for those discussed here, but rather the rival system also mentioned by Aristoxenus. What we are observing here, may thus be not just a more or less fixed pipe design oriented towards one of those late-Classical systems, but at the same time a potential enhancement of its musical capabilities that aulos makers would have applied to instruments from either *trópoi* system, on special request of their customers.

In the above figures, the absolute pitches of ancient Greek notes and scales have varied slightly, in order to reflect what appeared optimal for the instrument under scrutiny. While we have started with a reference pitch close to the standard of 490 Hz, Pipes 019 and 026 have suggested lowering it by about 30–40 cents. This does not negatively affect the tuning of Pipe 036 either. On the contrary, if that pipe indeed included 014 as its original bulb, a somewhat lower pitch results in a more convincing reed length. The precise values depend on the original length of the reed socket of 014, of which only 3.75 mm survive. The socket of 039 measures 20 mm, that of 017 about 17.5 mm (it does not terminate in a clear step); consequently we may estimate the missing length of 014 as about 15 mm. The high-pitched variant shown in Figure 15 would thus leave room for an effective reed length beyond the physical upper end of the pipe of only 2 cm (3.47 mm – 15 mm). Once the pitch is optimised for congruence with Pipes 019 and 026, this length increases to about 2.5 cm, which corresponds to a typical blade size of modern experimental aulos-reed reconstructions.²⁴ The difference between both scenarios amounts to 30–40 cents, the sixth part of a tone.

4.3.3 Bass notes of Pipes 019 and 026

Above, we have remarked on the identical length of the two pipes from the mouth end to the rims of the lowest preserved sections, without the spigots – which however are of strangely dissimilar dimensions, 15 mm and 25 mm, the latter being quite unusually long, and not only within the Oxus find. Beneath the little-finger hole, as we have seen, tubing without any further holes extends beyond the position of the bass hole of Pipe 036, and a comparison with the other instruments suggested that the lost part was too short to feature further toneholes. What then was the bass note of these pipes? Any further extension would take the pitch, tonehole or not, beyond Dorian $hyp\acute{a}t\bar{e}$ ΩM . The system of $tr\acute{o}poi$ offers only one lower note, Dorian $hyperyp\acute{a}t\bar{e}$, a whole tone below $hyp\acute{a}t\bar{e}$. Even though it forms a fifth with $m\acute{e}s\bar{e}$ ΩD , which was probably realised on the corresponding left-

²⁴ For an assortment of such reeds, cf. Wysłucha and Hagel 2023: 391 Fig. 17.

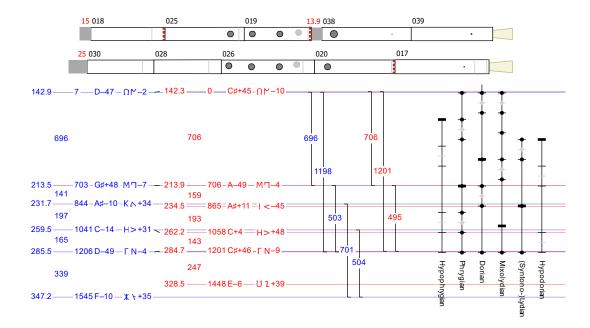


Figure 30: Pipes 019 (reed 26.0 mm) and 026 (reed 30.9 mm). Holes below: open, thumbholes closed; intervals ±20 cents; reference pitch: 480 Hz (35 cents below 490 Hz). Image by S. Hagel.

hand pipes, it stands in dissonant relationships of a second and a ninth with the boundary notes of the 'Dorian octave' and can therefore hardly be regarded as a very plausible bass note.

On closer inspection, the pipes have yet another surprise in store. If we do not append anything at all to their terminating spigots, the pitches emitted from these are indistinguishable, in spite of their diverse lengths (Figure 30). Moreover, they form a perfect octave with the top note of the system, ΓN , consequently a perfect fifth with the little-finger holes, and thus represent the Dorian $hyp\acute{a}t\bar{e}$ (which is at the same time the lowest note of the Phrygian and Mixolydian scales, expressed by the final letter of the alphabet, Ω , in ancient vocal notation. All this cannot possibly be written off as a coincidence.

Fortunately, the single explanation that offers itself can also account for the odd spigots, extra-long on one pipe, but unusually short on the other. It combines the most straightforward way of tuning the bass note during manufacture with, probably, an aesthetic motivation. As our calculations show, the length of a pipe alone does not sufficiently predict its pitch whenever there are fingerholes, or when the bore is not entirely cylindrical throughout. Indeed the parts of both pipes appear to have somewhat varying bores – for instance, 030 is wider than the rest – which make a clear difference. With perfectly cylindrical tubes, the bass notes would not coincide but differ by a sixth of a tone. Without the means to calculate the ensuing pitches, ancient makers could tune the pipe exits only by either changing their length or, somewhat, by enlarging the diameter at the aperture. In both cases, it was easy to take material away, but not to add anything. Consequently, one would have made the pipe a bit longer than expected, in order to shorten it progressively until

²⁵ On the significance of the projection of the alphabet onto early pitch systems, cf. Hagel 2009: 386–87.

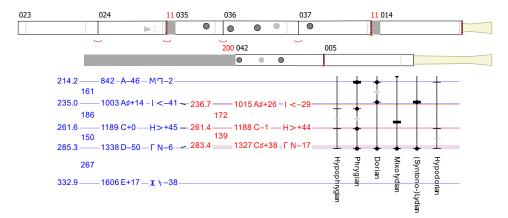


Figure 31: Pipes 036 (reed 38.7 mm) and 042 (reed 103.9 mm). Holes below: open, thumbholes closed; reference pitch: 480 Hz (35 cents below 490 Hz). Image by S. Hagel.

it sounded the desired pitch. Evidently, this would have been done before shaping the typical flare on the exterior surface.

In the present examples, the makers seem to have started with very long spigots. In the case of 030, they did not need to reduce it much, while 018 required cutting off a centimetre more. In the end, a custom-made end-piece must have been fitted over the remaining spigot, flush both with its end and with the surface of the tubing, but almost certainly flaring towards the exit. Any material would have been possible, but bone appears most likely – after all, the spigots show no traces of discolouration. This construction method would have had the big advantage of offering greater freedom regarding the shape of the 'bell'. Finding a bone with a suitable internal diameter that would in addition be thick enough at one end to be made into a bell may have been a difficult task. The aesthetics of item 008, for instance, which flares only minimally, is compromised by a natural groove in the bone, apparently a typical source of headache for aulos makers: we find a similar groove on one of the Megara pipes, where it was probably camouflaged.²⁶ By fitting a separate part over a spigot, this could simply be produced from a wider bone, or even made of ivory. In this way, not only the perfect pitch of the spigot exits and their unusual size find an explanation, but also the fact that two end parts seem to be missing. The small 'bells' may have crumbled to indistinct bone fragments, they may not have been recognised as belonging to the instruments even if intact, or, finally, they may even have been made from a less durable material.

4.3.4 Item 042

The lone three fingerholes from section 042, where the distance between index and thumb hole is smaller than that from thumb to middle finger, are incompatible with the same pattern. It may therefore come from a totally different kind of instrument, as may also be suggested by its much smaller diameter. Or it might have belonged to a left-hand pipe whose fingerholes were typically one step lower than those for the right hand (Figure 31).

²⁶ Terzēs and Hagel 2022: 25 n. 14.

5 A ritual instrument? (012+001)

5.1 Pitches

But not all pieces conform to the same scheme. Pipe 012, in particular, does not agree with the fingerhole patterns of the other right-hand pipes. If its upper two toneholes are musically aligned with those of Pipe 036, for instance, as is nicely possible with the proposed index hole from part 040, the lower three are increasingly off, and very few resonant intervals emerge (Figure 32). Here, at least, we must be dealing with a very different instrument.

From Figure 32, it also transpires that, if 040 is the correct index-hole part, the required reed stem would be much longer than on the other instruments. In contrast, the automated optimisation for this pipe alone in its proposed reconstruction suggests an effective reed length of just 3.3 cm, perfectly in line with the rest (Figure 33, where the abscissa indicates total lengths from pipe end to effective reed tip), producing five consonances within a threshold of 20 cents. Three of these however involve part 040, which, as we have seen, might alternatively belong on Pipe 001.

We have understood above that, unless Pipe 012 can be paired with Pipe 001, we would be forced to assume the complete loss of two more entire pipes – notably, two pipes whose counterparts appear quite well preserved. Being left-handed, 001 has no possible analogue for immediate comparison: of the two other presumable left-hand pipes, 009 has its unique mechanism, while item 042 covers a different set of fingerholes. As a consequence, we must evaluate Pipe 001 entirely on its own, just like 012.

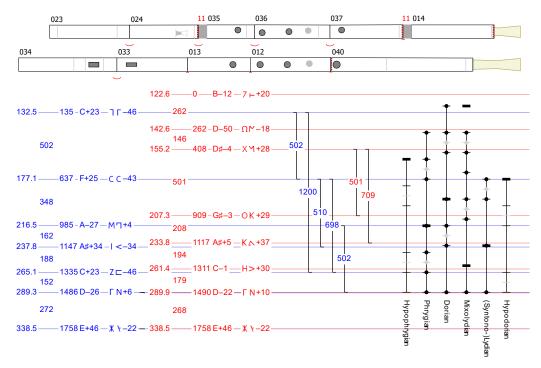


Figure 32: Pipes 036 (reed 34.7 mm) and 012 with part 040 (reed 62.8 mm). Holes below: open, thumbholes closed; intervals ±20 cents; reference pitch: 484 Hz. Image by S. Hagel.

For methodological symmetry, we must also begin its analysis with part 040 attached to its top and providing the index hole. At any rate, even if 040 was not physically the original part, the comparatively large distance between thumbhole and upper rim of 001 suggests that the index hole must have sat close to this rim, just as it does on 040, so that the latter must at least form a relatively good substitute. The same argument holds for 040 above 012, especially considering the formidable finger span this part already requires. For the resulting configuration of Pipe 001, the harmonicity optimum lies at 10.9 cm, with six acceptable consonances (Figure 34), once more two of which involve the index finger. Such an effective 'reed length' is clearly too large even for a very long reed stem, but at the same

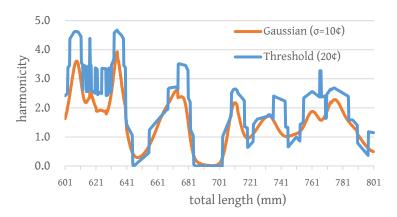


Figure 33: Harmonicity of Pipe 012 with part 040 as a function of effective total length in mm. Image by S. Hagel.

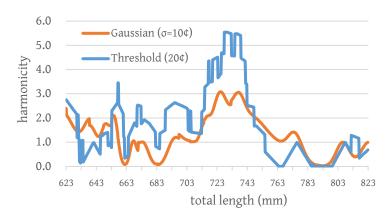


Figure 34: Harmonicity of Pipe 001 with part 040 as a function of effective total length in mm. Image by S. Hagel.

time too short for any of the extant mouthpiece sections plus reed. Consequently, if 040 belonged to either of these two pipes, which is quite probable, this pipe cannot have sported a bulb. It is still conceivable that another, shorter extension section above 040 on Pipe 001 bridged the gap to the reed, but overall it must be considered much more likely that 040 belongs with 012, after all, where it provides precisely the required length for a typically-sized reed.

With evidence as shaky as this – even if most of the joints appear quite plausible within themselves, all the small insecurities add up to a considerable product – we must be especially careful to establish as strong a methodology as possible. What we need is, essentially, independent corroboration. Fortunately we can obtain this by exploiting precisely the double nature of doublepipes, which defines three independent harmonicity optima: on the one hand, those based on the intervals of each pipe separately, on the other, that between the two pipes. The latter produces a pair of optimal effective reed lengths, just as the former do when taken together. In the more typical cases of evaluating an archaeologically well-defined pair of pipes, it normally suffices to look only at the sum of the three optima. In our case, it will be beneficial to separate them.

The algorithm used in Figure 33 and Figure 34 yields optima for each pipe separately, which are found around 63.5 cm and 72.7 cm, respectively. The smooth curves, created by addition of

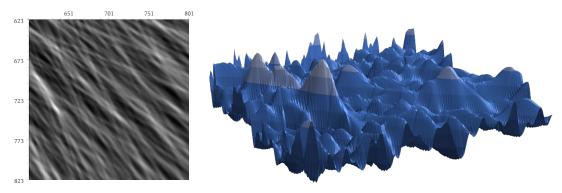


Figure 35: Harmonicity between Pipes 001 (y-axis) and 012 (x-axis), both with index-hole part 040, as a colour map and an elevation chart within the surveyed 20 cm×20 cm space (Cf. Hagel 2021: 414–15 with Fig. 9). Gaussian (σ =10 cents), lengths in mm. Image by S. Hagel.

	Pipe 012	Pipe 001	consonances (20 cents tolerance)
Individual evaluation			
threshold (20 cents)	63.53 cm	72.74 cm	5+6
Gaussian σ=10 cents	63.53 cm	72.57 cm	5+5
Between pipes only			
threshold (20 cents)	63.44 cm	73.24 cm	16+5+5
Gaussian σ=10 cents	63.43 cm	73.24 cm	16+5+5
Combined			
threshold (20 cents)	63.40 cm	73.19 cm 16+5+6	
Gaussian σ=10 cents	63.51 cm	73.34 cm 15+5+5	

Table 1: Optimal total effective lengths of Pipes 012 and 001, both with item 040, individually and in combination.

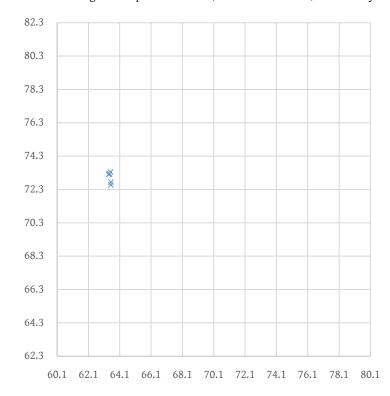


Figure 36: Clustering of length optima (Table 1) within the surveyed 20 cm×20 cm space. x-axis: Pipe 012; y-axis: Pipe 001. Image by S. Hagel.

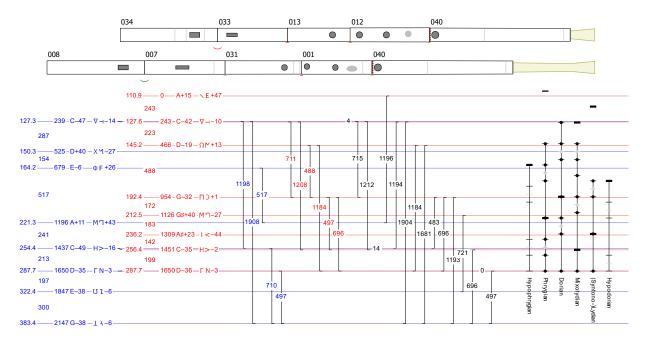


Figure 37: Pipes 012 (effective length 63.40 cm) and 001 (effective length 73.19 cm). Holes below: open, thumbholes closed; intervals ±20 cents; reference pitch: 484 Hz (490 Hz – 23 cents); harmonicity: 16/5/6. Image by S. Hagel.

Gaussian bells, and the alternative ragged threshold approach disagree by only a couple of millimetres, and only in the case of Pipe 001, whereas they fall together for Pipe 012.²⁷ The reason for the divergence is that Pipe 001 is not associated with a single sharply defined maximum at all; instead we find twin peaks, which imply a range of several millimetres of very similar harmonicity.

These lengths can be compared with an optimisation of intervallic relationships between the two pipes, excluding consonances within each (Figure 35). The various resulting numbers are detailed in Table 1 for comparison. It emerges that, although the two approaches are entirely independent, they predict identical optima. Figure 36, which also includes the optima for a combined evaluation, visualises the clustering, which is impeccable regarding 012, where all values stay within 1.3 mm, but naturally spread out a little bit more along the double maximum of 001, with 6.67 mm difference between pipe-internal and inter-pipe evaluation. It is difficult to see how such an excellent coincidence might be accounted for other than by assuming that the two pipes belong together – or at least that they formed part of largely identical instruments, which is equivalent for most purposes of our argument.

Figure 37, finally, displays the predicted pitches and intervals for the common optimum. It becomes clear that the top note of 001, for which a clone of item 040 still substitutes here, cannot possibly have duplicated a thumb-hole note from 012, whose position would be plainly out of the finger's reach. Instead, the fingering positions of the two pipes were apparently shifted by two holes. This is another important novelty, because so far only shifts of either a single hole were documented, including all early instruments, 28 or one of three holes, on the wooden auloi in the

For the methods and their robustness, cf. Hagel 2021: 413–15.

²⁸ Cf. e.g. Psaroudakēs 2008; Psaroudakēs 2014; Hagel 2020.

Louvre and the Egyptian Museum Berlin.²⁹ Though an offset of two holes had been predicted on the basis of an ancient text,³⁰ no corresponding material evidence has so far surfaced.³¹

At this point, we must return to the question of the index-finger hole of the lower pipe, 001. Originally we defined its position by using a duplicate of item 040, in the

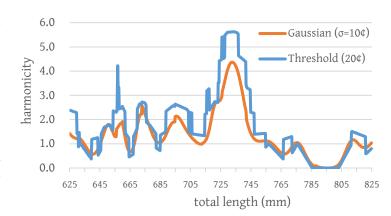


Figure 38: Harmonicity of Pipe 001 with corrected index hole as a function of effective total length in mm. Image by S. Hagel.

interest of a balanced methodology. By now we have however learnt that this part must rightly belong to Pipe 012. Nonetheless, as discussed above, the original pitch can hardly have been any lower because the hole in 040 seems drilled as close to the rim as possible, nor can it have been

much higher because of the limitations of the human hand. Even so, we cannot expect that the hole position (and size) of 040 reflected the original hole precisely and consequently need to ponder the consequences of small adjustments. With some virtual experimenting, 32 it emerges that the index hole could actually be drilled Table 2: up to half a centimetre higher

	Pipe 012	Pipe 001	consonances (20 cents tolerance)
Individual evaluation			
threshold (20 cents)	63.53 cm	73.35 cm	5+6
Gaussian σ=10 cents	63.53 cm	73.22 cm	5+5
Between pipes only			
threshold (20 cents)	63.46 cm	73.32 cm	16+5+6
Gaussian σ=10 cents	63.50 cm	73.38 cm	16+5+6
Combined			
threshold (20 cents)	63.52 cm	73.38 cm	16+5+6
Gaussian σ=10 cents	63.51 cm	73.39 cm	16+5+6

Table 2: Optimal total effective lengths of Pipes 012 and 001 with corrected index hole, individually and in combination.

without harming the instrument's harmonicity much. The optimal place, though, appears to be merely 2 mm higher, which makes its pitch coincide with its counterpart from the other pipe.

It is instructive to redo the automated optimisation algorithms with such a minimally corrected value. In this way, we can observe that the general interpretation is not affected by a small shift in the measurements, but that such a shift may still blur the image considerably regarding matters of detail. First of all, the internal harmonicity of the corrected Pipe 001 has now lost the annoying two peaks in favour of a clear-cut stronger single optimum (Figure 38).

²⁹ Cf. Bélis 1984; Hagel 2004; Hagel 2010; Hagel 2014.

³⁰ Hagel 2009: 408-10.

³¹ This may change with a new analysis of the Meroë find whose publication is expected shortly.

³² Essentially, I have increased the distance between index and thumb holes in steps of 1 mm and calculated the consequences. Differences smaller than a millimetre can confidently be neglected. Variations in hole size play a role in the physical design, but not for our present purposes because they can, in the given context of the highest tonehole, always be emulated by variation in position without noticeably affecting the resulting scales.

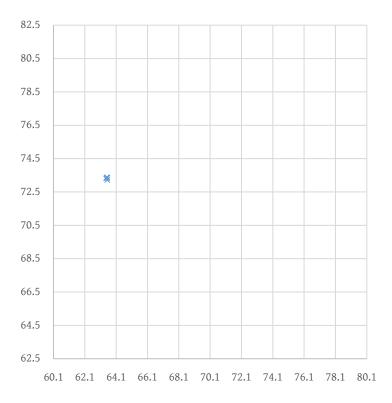


Figure 39: Clustering of length optima within the surveyed 20 cm×20 cm space (Table 2). x-axis: Pipe 012; y-axis: Pipe 001 with corrected index hole. Image by S. Hagel.

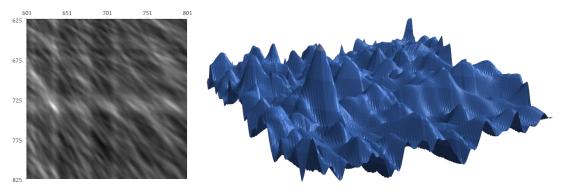


Figure 40: Harmonicity between Pipes 001 with corrected index hole (y-axis) + 012 (x-axis), as a colour map and an elevation chart within the surveyed 20 cm×20 cm space. Gaussian (σ =10 cents), lengths in mm. Image by S. Hagel.

The total lengths that yield maximum harmonicity between the two pipes are now perfectly close to those predicted by the evaluations of the individual pipes, with entirely negligible differences of less than a millimetre for Pipe 12 and less than two millimetres for Pipe 001 (Table 2). In the graphical representation, this convergence to a single point of desired total acoustical lengths becomes especially obvious (Figure 39). This optimum is not only better defined in terms of the sharpness of its peak, but it also stands out more clearly among the general noise of chance consonances in other configurations (Figure 40).

Finally, we obtain the just slightly adjusted set of pitches and intervals shown in Figure 41. It is important to note that nothing substantial has changed here – certainly nothing that might possibly affect a musical interpretation of the instrument. Nonetheless the astounding differences in

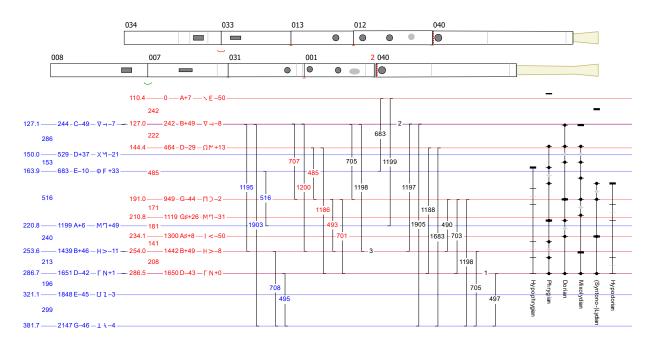


Figure 41: Pipes 012 (effective length 63.52 cm) and 001 (effective length 73.38 cm). Holes below: open, thumbholes closed; intervals ±20 cents; reference pitch: 482 Hz (490 Hz – 30 cents); harmonicity: 16/5/6. Image by S. Hagel.

the harmonic balance, which surfaced only by comparing calculated optima between an a-priori assumption and its subsequent minute adjustment, warrant some confidence in the reconstruction of the intended position of the lost index-finger hole.

Coming back to the lost index-finger hole on Pipe 001, we have seen that we are apparently able to predict its optimal and therefore likely its desired acoustical characteristics, but cannot translate these to precise physical parameters because variations in hole position along the tube and open hole area produce similar results. The historical truth may be still more complex. Apparently the missing section was none

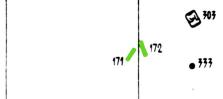


Figure 42: Items 001 ("171") and 002 ("172") on the excavation map. Image by G. Lindström, from a copy in the Drujinina archive.

other than 002, which we have above contemplated as the only potential but certainly not a perfect candidate for Pipe 009. Physically, 001 and 002 appear good matches; in addition, both feature enlarged fingerholes, and also a rim above the shallow recess over their sockets, which apparently prevented an enforcing winding of thread from coming off the edge. Anyway, the case may already appear settled by their find spots, immediately next to each other (though misleadingly separated by the border between quadrants 14 and 15; cf. Figure 42).

With its slightly elongated hole, obviously designed to produce at least two distinct pitches, 002 does not lend itself to a precise calculation of the lower one and therefore defies a mathematical corroboration of our prediction. Incidentally, though, its pitch (and all the others) are practically identical with those predicted when it is precisely half open (at a total effective pipe length of 73.24 cm). When fully opened, though, the index-finger hole sounds a fourth above the little finger of the right-hand pipe, a hole that otherwise stands in consonant relation only to the exit of Pipe 001. This may sufficiently explain the curious shape of the instrument's treble hole.

5.2 Scales

Can we understand the pitches in terms of transmitted music theory? Actually, the same system of *trópoi* that appeared to be reflected in the other Oxus instruments may hold the key here as well. Set to the same pitch level as the other pipes suggested, the highest note from the lower, longer, left pipe once more plays the highest note of the system, Γ N (cf. Figure 41). The note of the right thumb was a tone higher, officially a fifth above Phrygian *mésē* or Dorian *paramésē* M Γ I. It corresponds to the ancient instrumental note Γ I, which plays such an important role in the Orestes papyrus, probably related to the aulos accompaniment of the tragic song. Above it, we might expect another whole tone, leading to the highest member of the graphical triplet N Γ I. Instead, we find a minor third, much as on Pipe 019 and one of the auloi from Megara. Once more, mastering such an extraordinary finger span is rewarded by access to the additional consonances of a fourth and a fifth, here with the two highest holes of the left pipe. Moreover, it will provide a great-sounding twelfth with the lower rectangular tonehole on Pipe 001, a perfect octave with its little-finger hole, and also an eleventh with the higher rectangular hole – an interval, whose status as a consonance was controversial in antiquity.

Looking at the individual scales, it seems quite clear that this is no Phrygian instrument. Even though the boundary notes of the Phrygian octave scale are present, which is just a corollary of the alignment of all scales at their upper end, and the second lowest note could almost certainly be accessed by down-tuning the higher rectangular hole of 012 by means of its preserved slider (Figure 3 and Figure 4), the other required pitches are ill-provided for. Conceivably, though, the elongated thumb hole on 001 is meant to give somewhat awkward access to a couple of Phrygian notes, perhaps rather for temporary modulation. In contrast, the instrument appears perfectly suited for the Mixolydian. Both pipes play its top note as well as its $m\acute{e}s\ddot{e}$, a tone lower. The $pykn\acute{o}n$ below is produced by the left ring finger, opening, half-stopping, and closing its hole. The next note, again below, is a diatonic intruder into the otherwise enharmonic scheme. Here it appears

³³ Pöhlmann and West 2001 no. 3; cf. West 1992b: 206-7.

These would be paramésē and trítē (diezeugménōn), while the highest note of the same pyknón, paranétē (diezeugménōn), would be much more difficult to play. But perhaps providing for diatonic Phrygian was considered sufficient: Aristoxenus reports that diatonic is specifically suited for this harmonía, just as enharmonic was most intimately associated with Dorian (Clement of Alexandria, Strom. 6.88.1) and the aulos Megara 1965Δ appears also ill-suited for enharmonic Phrygian (Terzēs and Hagel 2022: 58).

The identity of the Mixolydian *mésē* had been subject to discussion and was defined differently in the two pre-Aristoxenian *trópoi* systems. While the other one located it in the lower part of the octave and this view is reflected in the ancient *harmoníai* transmitted by Aristides, whose notes are indicated by circles in the above diagrams, the system on which the Oxus instruments are based recognised it as situated a whole tone below the treble note (cf. West 1992b: 223–24; Hagel 2000: 172–73). This redefinition obviously demanded having that note available on the instrument. The Megara auloi are representatives of the alternative system, which placed the Mixolydian a semitone higher (Terzēs and Hagel 2022: 48–60).

It is not absolutely certain whether this note, which was analysed as the Mixolydian $m\acute{e}s\bar{e}$ in the rival system, was required at all in the present one (see the preceding note). However, since it must have substantially contributed to the modal character of the scale, it is unlikely to have been abandoned altogether.

in a soft-diatonic variant, about five quartertones below instead of four, which was very common at the start of the Hellenistic period, according to Aristoxenus.³⁷

Half-closing the same note by the slider attached to it, which must have been possible during playing, would give the highest note of the lower *pyknón*; closing it completely, the middle note, while the lowest would have been directly provided for by the higher bass hole of the left pipe, when completely opened. The same *pyknón* also forms part of the Dorian, whose $m\acute{e}s\bar{e}$ Π is played by the left little finger. The note above, Dorian *paramésē* $M\Pi$, falls short of the disjunctive tone required by theory. Being more in the range of what modern music theory calls a 'minor whole tone', one might wonder whether it was perhaps drilled as a compromise between the Dorian note and the somewhat lower Mixolydian *oxýpyknon* (written NX in the canonical notation system).

We have encountered precisely the same deviation from Aristoxenian theory when analysing the implications of part 009+043 (above, chapter 3.2). There the lowered Dorian paramésē may have seemed a deplorable corollary of the modulating mechanism. The present instrument, however, is not restrained in such a way, and its makers were free to drill the hole in question higher up. They were also not answering to physical constraints – a higher ring-finger hole would actually be easier to play. Nor does the compromise theory hold much value: it would be relatively easy to lower the pitch by the required quartertone by moving the finger over the rim of the hole, while raising it implies manipulation of the reed mouthpiece, which risks losing the basic harmonic balance. However, whereas we needed to interpret 009+043 mostly on the basis of a single section, in the case of Pipe 001, the presence of adjacent sections and, most importantly, the counterpart pipe allows us to assess the phenomenon of the lowered Dorian paramésē in its wider musical context. It so emerges that the 'wrong' pitch in fact establishes a nicely pure minor third with the right ring finger (calculated as 319 cents, which is for all practical purposes indistinguishable from a mathematically perfect third's 316 cents).

No ancient authority, of course, has ever described thirds as consonances. Nonetheless, modern scholars have sought to interpret Aristoxenus' concept of 'sweetening' in terms of diminishing harsh 'Pythagorean' to resonant pure major thirds. But there is also a direct testimony to the use of simultaneous thirds, ultimately coming from the pen of Aristoxenus himself. In another passage quoted by pseudo-Plutarch (*Mus.* 1137c), Aristoxenus lists aulos notes used only in the accompaniment and specifies the melodic notes with which they would go. The resulting intervals are all different; they include a fifth and a fourth, but also a major and a minor third, a minor sixth and

Ps.-Plut. Mus. 1145d: μαλάττουσι γὰρ αἰεὶ τάς τε λιχανοὺς καὶ τὰς παρανήτας "they always soften the likhanoí and paranêtai". The note in question would be analysed as one of those, specifically as paranêtē diezeugménōn in the abstract scheme of the Greater Perfect System, or as likhanós in a lyre-based approach. For a soft diatonic on the Megara finds, cf. Terzēs and Hagel 2022: 53–54. For a general overview of tetrachord divisions, see e.g. West 1992b: 166–72.

³⁸ Winnington-Ingram 1932: 200; Barker 1989: 50; Barker 2000: 122; Franklin 2005: 26–28.

For a discussion of the passage and its relation to aulos music, as well as a diagram of the note pairs in question, cf. Hagel 2009: 407–11. Note that the notation signs that are there attached to the pitches of the Dorian scale reflect a hypothetical earlier stage of notation use and are therefore different from those used here, which derive from the late-Classical development of modulating diagrams.

even a major second. For our purposes, the minor third is of special interest. It is realised between $n\acute{e}t\~{e}$ synēmménōn and paramésē, precisely the notes that bound the pure third on our Oxus aulos in the Dorian key. Moreover, the music in question is called the 'libation style' (spondeiakós trópos), which was evidently associated with the Dorian mode and the enharmonic genus, just as the 'libation tune' (tò spondeîon) was said to have been composed on a Dorian aulos. ⁴⁰ In short, the only reconstructible aulos pair from the Oxus temple produces a pure third precisely between those two Dorian notes which Aristoxenus attests were played simultaneously in aulos music related to sacrificial ceremonies.

That our aulos is above all a Dorian instrument seems further corroborated by the lowest note that can be sounded on both pipes, which corresponds to the lowest note in the Dorian *harmonía* as transmitted by Aristides Quintilianus. In this way, the overlapping range of the two pipes is identical with the extent of the Dorian scale.⁴¹ Further corroboration is gained from the two-hole offset between the pipes, which was predicted precisely for Dorian instruments.

In the bass region, not all notes of the Dorian harmonía can be accessed at the same time. A composer would have to decide which combinations to use for a specific piece, or a specific section in a larger composition. Ancient theory reflects this organological fact in a concept that Aristides Quintilianus calls pétteia (1.12, p.29.18-21 Winnington-Ingram), a designation apparently taken from a strategic board game. Each of the pipes has two rectangular bass holes, of which the higher is longer and narrower, the lower, wide and short. Generally, the latter design is more useful for quickly stopping or releasing the entire opening, the former rather for gradual adjustment, accessing intermediate pitches as well. In our specific case, however, the distinction may rather be between the different types of mechanism. On Pipe 012, where these are preserved, the short wide hole is operated by the collar, which needs to be adjusted before a performance, while the slider over the longer hole could probably be operated during performance. The similar design on Pipe 001 suggests that it was equipped with an analogous pair of different mechanisms. While the collar hole on item 008 almost certainly required no half-closing because its note is isolated, the same may not have been true for its counterpart on 034, which might have been partially opened in order to play the Dorian hypáte mesôn Ω ^{Γ} when the hole for this note on the other pipe was closed, in order to give access to the 'irregular' note that extended the 'proper' Dorian octave down to $\nabla \rightarrow$, expanding it to the ninth that Aristides attributes to the Dorian harmonía. Regarding the longer holes, we have already discussed how the slider on item 033 would switch between the 'irregular' Mixolydian note and the oxýpyknon below. In contrast, the trópoi system includes no note close below the Dorian *hypátē*, to which the presumable slider on 007 might have retuned its hole.

Ps.-Plut. Mus. 1135a. For other relevant passages, see Hagel 2009: 394 n. 64.

The fact that the note from the lower end of 034 coincides so exactly with that from the last tonehole of Pipe 001 seems to prove that this was indeed the exit of the pipe. The apparent socket there must therefore have been but a counterbore, intended to affect the sound of the bass note. Anyway, featuring a step of hardly a millimetre, the connection would have been precarious with any non-metallic spigot.

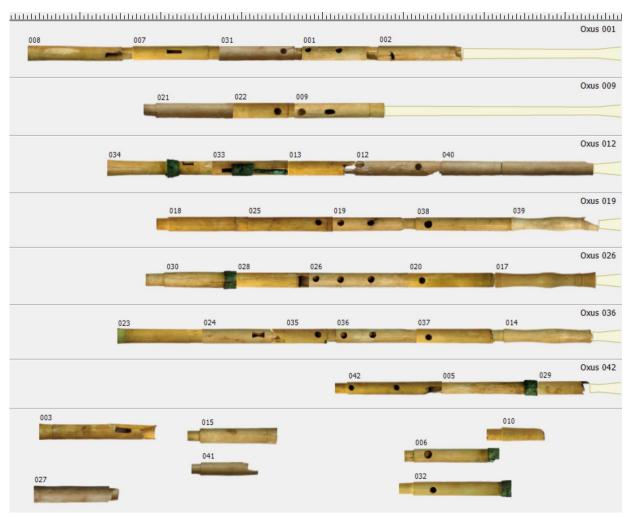


Figure 43: As Figure 23, but with photographs instead of technical sketches. Screenshot from the software used for virtual experimentation. Note that the relative rotation of the pieces is arbitrary. Image by S. Hagel.

Apart from the Dorian, we have seen that the instrument also covers the Mixolydian. This recalls yet another passage quoted from Aristoxenus (ps.-Plut. *Mus.* 1136d), which attributes the 'coupling' of these two modes to the tragedians, that is, composers of particular styles of aulos-accompanied song. The Megara auloi have been interpreted as technically reflecting this very act of 'coupling' within the paradigm of the 'commensurable' *trópoi*;⁴² the Oxus instrument seems to contribute analogous evidence from the rival system.

All this may also teach us to view the famous statuette of an aulos-playing Marsyas that was dedicated to the temple in much sharper focus. After all, the libation tune, like other ritual music, ⁴³ was attributed to the legendary aulete Olympus, who in turn derived his art from no other than Marsyas himself, his master and lover. Whoever played the aulos for the rites and sacrifices conducted at the Oxus temple would therefore trace their artistic genealogy back to Marsyas. A statuette of the Satyr would thus become as apt a dedication to the god during a player's career as would

⁴² Terzēs and Hagel 2022: 49-50.

⁴³ Ps.-Plut. Mus. 1133d-e.

be the sacrificial aulos itself on retirement.⁴⁴ Most appropriately this would have been a Dorian, enharmonic instrument, firstly, because this was the style of traditional sacrificial music, but also, because the Dorian would doubtless suit a divinity with martial connotations best, such as seem to have been attributed to the River Oxus.⁴⁵

6 Reed lengths

The preceding attempts at reconstruction are once more visualised in Figure 43. In five cases, concerning Pipes 012, 019, 026, 036, and, with some caution, 042, they have produced both a physical proximal end and an optimal effective total acoustic length and therefore, as a sort of side product of automatic optimisation, a computer-generated assessment of the effective lengths of the reed mouthpieces outside their sockets (note that this is only an approximate value for 036 and 042, where the original depth of the broken socket needed to be estimated 46).

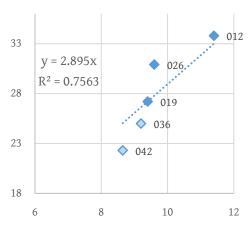


Figure 44: Effective reed length (y-axis) as a function of bore diameter (x-axis), in mm.

Best-fit line for strict correlation. Image by S. Hagel.

These lengths are roughly consistent, ranging from 25 mm to 33.8 mm. On closer inspection, they increase with the diameter of the bore, as shown in Figure 44. The correlation is strong enough to explain 75% of the data, even under the constraint of a strictly proportional relationship (i.e., of the form $l=k\cdot d_i$).

This observation finds a straightforward explanation in the fact that mouthpieces for instruments with wider bore were naturally made from thicker reed stalks. Only by matching their internal diameter with that of the tube one could get the largest possible intervals out of the limited resource of finger spans: reeds with reduced diameter tend to flatten the highest notes. Wider stems, in turn, lead to wider reed blades in the process of flattening the reed tips – which must at the same time be proportionally longer, if an optimal reed geometry is to be maintained. As a rule of thumb, the Oxus reeds appear to have extended from their sockets by about three times the bore diameter. This, of course, may reflect only the practices of a single workshop. Otherwise we have evidence for Hellenistic bone pipes with sliders only from one of the Megara pipes, where the reed is generally comparable, if a bit longer, with a bore-to-reed-length ratio of 27%, as opposed to the 31% to 37% for the Oxus instruments. We must wait for future research and more finds.

It may be important to remark that, while bore diameter is a good indicator of reed length, socket diameters are not (Figure 45). This suggests that the reed diameter was indeed selected ex-

⁴⁴ Cf. Anth. Pal. 5.206; 6.82; 6.94.

⁴⁵ Cf. the notorious description of the Dorian mode as reflecting the 'sound' of males at war or otherwise toiling in Plato's *Republic* 399a–c, and the wide-spread opinion that the enharmonic would make people courageous cited (and duly criticised) in Pap. Hibeh 13.13–21 (cf. West 1992a: 16–23).

⁴⁶ An estimate based on the average of extrapolations from the three complete sockets scaled by bore diameters suggests a socket depth of about 22.3 mm for 029 on Pipe 042. For 014 on Pipe 036, cf. above, chapter 0.

clusively for musical suitability, while variations in the size of the gap between reed stem and socket wall, which was doubtless sealed with a winding of waxed thread, were of no consequence.

Estimated losses

Our reconstructions, including those of plausible bass notes for the two instruments with only seemingly lacking tail pieces, finally allow us a better-informed estimate of the minimal total losses. Starting from surviving exits and upper lengths that are either extant or have been assessed by calculating plausible scales, we obtain good data for five pipes, 001, 012, 019, 026, and 036. Conservative guesses are possible for 009, supposing it played the same bass note as 001, the only left-hand pipe whose lower end we have, which seems the first harmonically meaningful pitch beyond the surviving length of 009, and for 042, assuming that it was not similarly long, but rather in the range of the expected left-hand counterparts of the $_{Figure\ 45:\ Effective\ reed\ length}$ (y-axis) and shorter right-hand pipes, 019 and 026. As left pipes, these would typically have been a bit longer, which suggests the lowest note of the system as their bass note, a whole tone



socket diameter (x-axis), in mm. Above: diameter at the deep end; below: diameter at the exit. Image by S. Hagel.

below the exit of 019 and 026. Adding the missing 'bell' pieces of the latter to the lost lengths, we find that about 21% of those pipes are missing of which we may have been able to reconstruct substantial parts (Table 3). Wherever these reconstructions may appear overly optimistic, the implied losses would increase. But we have parts of at least two more instruments. In order to minimise the number of assumed missing instruments, we need to attribute these hypothetically to two left pipes, and in order to minimise the amount of missing material, to as short left pipes as is plausible. This leads us to the presumed extent of 042. The two pipes would thus complement 019 and 036, which leaves us with one stray fingerhole section, 006, to which we can only assign the shortest extant length otherwise attested, that of 019. On the basis of the hypothesis that the temple stored individual pipes rather than whole instruments, this would entail a loss of at least 34%, a third of the material. Starting from the more plausible assumption that what we have are the remnants of complete doublepipes, we need to assume four additional lost pipes, those respectively complementing Pipes 009, 026, 042, and the one from which 006 survives. The most conservative estimates here involve the lengths of 012 (paired with 001, the other long left pipe), twice 019 (once more, the shortest right pipe), and finally 042 (the short left pipe, since we have before made 006 part of a right-hand pipe). The estimated losses thus rise to 45%, almost half of what

	Pipe	hand	length	
Pipes with extant lengths	001	L	700.6mm	
	012	R	601.5mm	
	019	R	547 . 6mm	
	026	R	557 . 2mm	
	036	R	603.5mm	
Pipes with estimated lengths	009	L	708.4mm	same bass note as 001
	042	L	608.8mm	lowest tonehole on 001 ⁴⁷
Additional missing pieces	019		14 . 9mm	
	026		25 . 0mm	
Expected on 7 pipes			4367.5mm	
Extant on 7 pipes			3435 . 0mm	
Losses from 7 pipes			21.4%	
Three more pipes inferred from stray parts	019-L (032)	L	608.8mm	estimated from 042
	036-L (010 003?)	L	608.8mm	estimated from 042
	? (006)	(R)	547 . 6mm	estimated from 012
Unattributed extant parts			589 . 6mm	including 94.4 mm from doubtful 027
Expected on 10 pipes			6132.8mm	
Extant			4024.6mm	
Losses from 10 pipes			34.4%	
Missing complementary pipes	009-R	R	601.5mm	estimated from 012
	026-L	L	547 . 6mm	estimated from 019
	042-R	R	547 . 6mm	estimated from 019
	?-(L)	(L)	608.8mm	estimated from 042
Expected for 6 instruments			7281.9mm	
Losses from 6 instruments			44.7%	

Table 3: Estimating the extent of losses.

there might once have been. These, it must be emphasised, are all minimal figures. I leave it to taphonomists and excavation historians to ponder what may have happened here.

8 Conclusions

The Oxus find contributes some crucial parts to the jigsaw of aulos science, in some respects apparently confirming predictions made on the basis of non-material evidence. Three of the reconstructed pipes appear quite similar, with the exception of the lower part of one, which included a specially-shaped tonehole. Together these highlight a musical tradition of Dorian music, presumably sacrificial, or possibly in other ways ritual. A pair of even longer pipes, equipped with metal appliances for closing bass holes, also appear to be centred on the Dorian <code>harmonía</code>, apparently with Mixolydian and perhaps Lydian capabilities. Most notably, if reconstructed correctly, it fur-

 $^{^{\}rm 47}$ Calculated to play the same pitch with two more fingerholes for 042.

nishes the first archaeological evidence for a Greek-style doublepipe where the finger positions of the hands are offset by two holes instead of one or three. One prima facie 'problematic' pitch appears to indicate the use of pure thirds, here intentionally reflected in instrument design.

Another pipe, finally, revealed a retuning mechanism consisting of two bone tubes, one of which could be turned inside the other, resizing the finger holes and thus enabling switching between scales.

All the instruments appear designed in view of the same integrated system of various *harmoníai*. Having been predicted on the basis of ancient texts many years ago, this system aligns all scales at their upper end, at the expense of placing some of them in mutual relations of an odd number of quartertones, defying modulation through the circle of fifths. The harmonic context of the find thus accords excellently with its presumed date: it is only to be expected that the mutually incompatible musical systems known to Aristoxenus in the late fourth century BCE continued to be employed by their respective adherents for at least some decades, being handed down from masters to students and upholding demand for suitable instruments.

The Oxus pipes appear to share a common pitch standard of about 480 Hz (relative to the later Lydian *hyperbolaía*, which is structurally equivalent to modern A_4), which is just about a third of a tone lower than the standard of 490 Hz that had been derived from other ancient instruments.

A certain flexibility in placing the treble fingerhole now appears to have been another traditional feature of aulos manufacture. Musicians who managed extreme finger stretches could thus avail themselves of additional concords, out of the reach of average players.

All in all, a broad picture of ritual instruments emerges whose features can be fully explained within our knowledge of ancient Greek music. Greek culture may of course have shared many of the relevant traits with other peoples, but at least the hardly disputable provision for quartertone intervals seems to point to a more intimate connection with Greece proper, as do many details that appear to confirm certain passages in ancient musical writings as well as predictions that had been based on these. With the various types of mechanisms, two of which are here documented for the first time, the music of the Oxus temple would hardly have fallen short of Mediterranean standards. Not all the discussed details may be assessed with the same degree of confidence – after all, we have barely more than half of the pieces that had originally formed at least six instruments. On the other hand, our reconstruction was repeatedly confirmed by independent evidence, such as reed lengths that all conform to the same principle, so that we may reasonably hope to have attained a general understanding of this unique find. Without doubt, it can help in understanding a crucial period in the development of sophisticated wind instruments, extending our knowledge of the Hellenised musical world chronologically, geographically, and not least culturally, as the deduced modes seem to chime in quite well with the find's religious context.

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